Chapter 13: Math and…

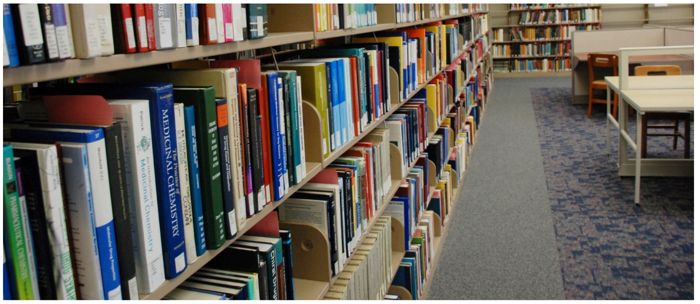

Figure 1: Math can be found in many different areas and subjects.

Where do we find math around us? Math can be found in areas that are expected and sometimes in areas that are surprising. There are many ways that mathematical concepts, such as those in this text, are infused in the world around us. In this chapter, we will explore a sampling of five distinct areas from everyday life where math's impact plays a meaningful role.

Math's impact on art can be found in numerical relationships that are known to create or enhance beauty. The Fibonacci numbers are one mathematical example that can be found in nature such as in petal count of a rose. On a different note, a mathematical exploration can aid in making a convincing argument on how we can positively impact our environment. Whether looking at the choices of a single individual or the larger impact offered from a collaborative effort, there are measurable responses to positively address climate change. Turning to medicine, which has been a topic of global importance in recent years, we will explore how math is used to determine drug dosage rates and test the validity of a vaccine. Switching back to items of aesthetic nature, we will examine some foundational components of music which, like art, brings beauty and joy to our lives. Finally, we will explore some ways that math is used in sports to predict future performance and analyze tournaments styles.

13.1 Math and Art

Figure 2: Sunflower seeds appear in a pattern that involves the Fibonacci sequence.

Learning Objectives

After completing this section, you should be able to:

1. Identify and describe the golden ratio.
2. Identify and describe the Fibonacci sequence and its application to nature.
3. Apply the golden ratio and the Fibonacci sequence relationship.
4. Identify and compute golden rectangles.

Art is the expression or application of human creative skill and imagination, typically in a visual form such as painting or sculpture, producing works to be appreciated primarily for their beauty or emotional power.

Oxford Dictionary

Art, like other disciplines, is an area that combines talent and experience with education. While not everyone considers themself skilled at creating art, there are mathematical relationships commonly found in artistic masterpieces that drive what is considered attractive to the eye. Nature is full of examples of these mathematical relationships.

Enroll in a cake decorating class and, when you learn how to create flowers out of icing, you will likely be directed as to the number of petals to use. Depending on the desired size of a rose flower, the recommendation for the number of petals to use is commonly 5, 8, or 13 petals. If learning to draw portraits, you may be surprised to learn that eyes are approximately halfway between the top of a person's head and their chin. Studying architecture, we find examples of buildings that contain golden rectangles and ratios that add to the beautifying of the design. The Parthenon , which was built around 400 BC, as well as modern-day structures such the Washington Monument are two examples containing these relationships. These seemingly unrelated examples and many more highlight mathematical relationships that we associate with beauty in artistic form.

Figure 3: The Parthenon in Greece demonstrates the golden ratio.

Golden Ratio

The **golden ratio**, also known as the golden proportion, is a ratio aspect that can be found in beauty from nature to human anatomy as well as in golden rectangles that are commonly found in building structures. The golden ratio is expressed in nature from plants to creatures such as the starfish, honeybees, seashells, and more. It is commonly noted by the Greek letter ϕ (pronounced "fee"). $\phi = \frac{1+\sqrt{5}}{2}$, which has a decimal value approximately equal to 1.618.

Consider : Note how the building is balanced in dimension and has a natural shape. The overall structure does not appear as if it is too wide or too tall in comparison to the other dimensions.

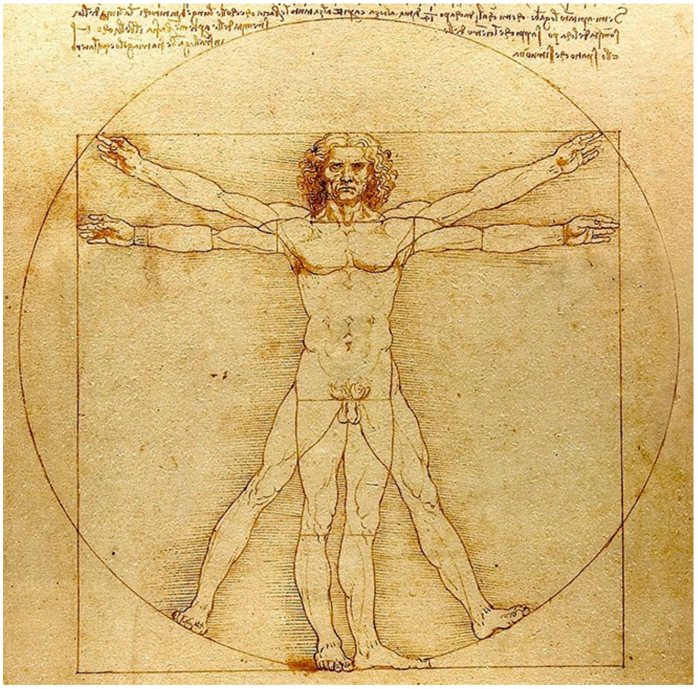

Figure 4: *Vitruvian Man* by Leonardo da Vinci

The golden ratio has been used by artists through the years and can be found in art dating back to 3000 BC. Leonardo da Vinci is considered one of the artists who mastered the mathematics of the golden ratio, which is prevalent in his artwork such as *Virtuvian Man* . This famous masterpiece highlights the golden ratio in the proportions of an ideal body shape.

The golden ratio is approximated in several physical measurements of the human body and parts exhibiting the golden ratio are simply called golden. The ratio of a person's height to the length from their belly button to the floor is ϕ or approximately 1.618. The bones in our fingers (excluding the thumb), are golden as they form a ratio that approximates ϕ. The human face also includes several ratios and those faces that are considered attractive commonly exhibit golden ratios.

Example 1 **Using Golden Ratio and a Person's Height**

If a person's height is 5 ft 6 in, what is the approximate length from their belly button to the floor rounded to the nearest inch, assuming the ratio is golden?

Solution

Step 1: Convert the height to inches

$$5 \text{ ft } 6 \text{ in} = 66 \text{ in}$$

Step 2: Calculate the length from the belly button to the floor, L.

$$\frac{66}{L} = 1.618$$
$$L = 40.8 \text{ in}$$

The length from the person's belly button to the floor would be approximately 41 in.

Fibonacci Sequence and Application to Nature

Figure 5: Rose petals appear in a Fibonacci spiral.

The **Fibonacci sequence** can be found occurring naturally in a wide array of elements in our environment from the number of petals on a rose flower to the spirals on a pine cone to the spines on a head of lettuce and more. The Fibonacci sequence can be found in artistic renderings of nature to develop aesthetically pleasing and realistic artistic creations such as in sculptures, paintings, landscape, building design, and more. It is the sequence of numbers beginning with 1, 1, and each subsequent term is the sum of the previous two terms in the sequence (1, 1, 2, 3, 5, 8, 13, ...).

The petal counts on some flowers are represented in the Fibonacci sequence. A daisy is sometimes associated with plucking petals to answer the question "They love me, they love me not." Interestingly, a daisy found growing wild typically contains 13, 21, or 34 petals and it is noted that these numbers are part of the Fibonacci sequence. The number of petals aligns with the spirals in the flower family.

Example 2 **Applying the Fibonacci Sequence to Rose Petals**

Suppose you were creating a rose out of icing, assuming a Fibonacci sequence in the petals, how many petals would be in the row following a row containing 13 petals?

Solution

The number of petals on a rose is often modeled with the numbers in the Fibonacci sequence, which is 1, 1, 2, 3, 5, 8, 13,..., where the next number in the sequence is the sum of $8 + 13 = 21$. There would be 21 petals on the next row of the icing rose.

Golden Ratio and the Fibonacci Sequence Relationship

Mathematicians for years have explored patterns and applications to the world around us and continue to do so today. One such pattern can be found in ratios of two adjacent terms of the Fibonacci sequence.

Recall that the Fibonacci sequence $= 1, 1, 3, 5, 8, 13, \ldots$ with 5 and 8 being one example of adjacent terms. When computing the ratio of the larger number to the preceding number such as 8/5 or 13/8, it is fascinating to find the golden ratio emerge. As larger numbers from the Fibonacci sequence are utilized in the ratio, the value more closely approaches ϕ, the golden ratio.

Example 3 **Finding Golden Ratio in Adjacent Fibonacci Terms**

The 24th Fibonacci number is 46,368 and the 25th is 75,025. Show that the ratio of the 25th and 24th Fibonacci numbers is approximately ϕ. Round your answer to the nearest thousandth.

Solution

$75,\frac{025}{46},368 = 1.618$ The ratio of the 25th and 24th term is approximately equal to the value of ϕ rounded to the nearest thousandth, 1.618.

Figure 6: The pyramids of Giza in Egypt

Golden Rectangles

Turning our attention to man-made elements, the golden ratio can be found in architecture and artwork dating back to the ancient pyramids in Egypt to modern-day buildings such as the UN headquarters. The ancient Greeks used **golden rectangles**—any rectangles where the ratio of the length to the width is the golden ratio—to create aesthetically pleasing as well as solid structures, with examples of the golden rectangle often being used multiple times in the same building such as the Parthenon, which is shown. Golden rectangles can be found in twentieth-century buildings as well, such as the Washington Monument.

Looking at another man-made element, artists paintings often contain golden rectangles. Well-known paintings such as Leonardo da Vinci's *The Last Supper* and the *Vitruvian Man* contain multiple golden rectangles as do many of da Vinci's masterpieces.

Whether framing a painting or designing a building, the golden rectangle has been widely utilized by artists and are considered to be the most visually pleasing rectangles.

Example 4 **Finding Golden Rectangle in Frames**

A frame has dimensions of 8 in by 6 in. Calculate the ratio of the sides rounded to the nearest thousandth and determine if the size approximates a golden rectangle.

Solution

$8/6 = 1.333$; A golden rectangle's ratio is approximately 1.618. The frame dimensions are close to a golden rectangle.

PEOPLE IN MATHEMATICS

M.C. Escher

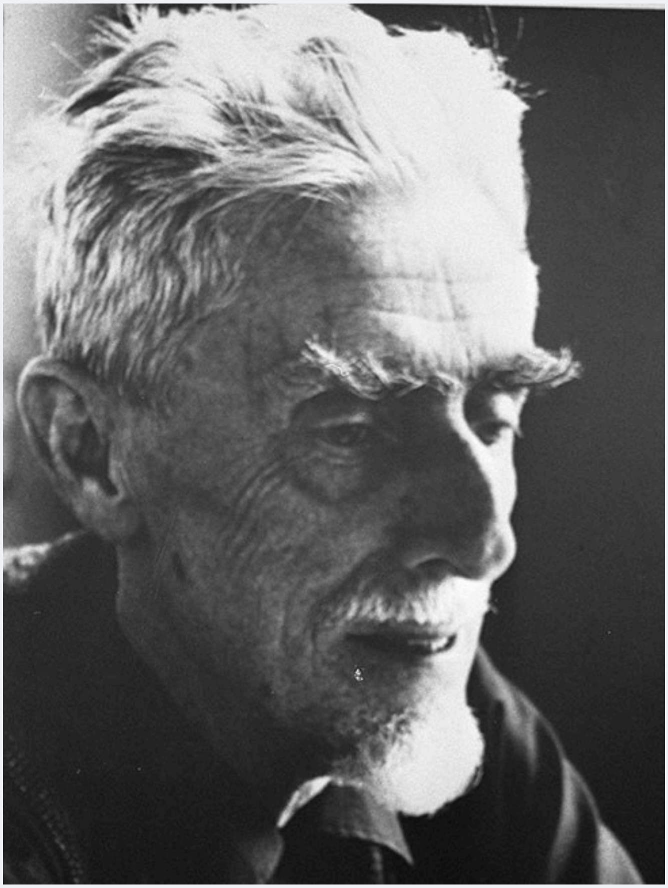

Figure 7: M.C. Escher

Mauritis Cornelis Escher was a Dutch-born world-famous graphic artist and his work can be found in murals, stamps, wallpaper designs, illustrations in books, and even carpets. Over his lifetime, M.C. Escher created hundreds of lithographs and wood engravings as well as more than 2,000 sketches.

Escher's work is characterized with the infusion of geometric designs that obey most of the mathematical rules. If you study his work closely, you can see where he breaks a mathematical relationship to create famous illusions such as soldiers marching around the top of a square turret where the soldiers appear to be always going uphill but are contained on a single set of stairs in a square. Look closely and the golden ratio as well as golden rectangles abound in Escher's work.

Like many famous people, M.C. Escher did not find success in his early school years. Before finding success, Escher failed his final school exam and quit a short stint in architecture. Finding a graphic arts teacher who recognized Escher's talent, Escher completed art school and enjoyed traveling through Italy, where he found much of his inspiration for his work.

Key Terms

- golden ratio
- ϕ
- Fibonacci sequence
- golden rectangle

Key Concepts

- The golden ratio, ϕ, can be found in nature, and the relationship is often associated with beauty and balance.
- The Fibonacci sequence reflects a pattern of numbers that can be found in various places in nature. The sequence can be used to predict other values that follow the Fibonacci pattern.
- State some naturally occurring applications of the Fibonacci sequence.
- State some naturally occurring applications of the golden ratio.
- Determine if a rectangle is golden.
- State some artistic applications of the golden rectangle.

13.2 Math and the Environment

Figure 8: Solar panels harness the sun's energy to power homes, businesses, and various methods of transportation.

Learning Objectives

After completing this section, you should be able to:

1. Compute how conserving water can positively impact climate change.
2. Discuss the history of solar energy.
3. Compute power needs for common devices in a home.
4. Explore advantages of solar power as it applies to home use.

   Climate change and emissions management have been debated topics in recent years. However, more and more people are recognizing the impacts that have resulted in temperature changes and are seeking timely and effective action. The World Meteorological Organization shared in a June 2021 publication that "2021 is a make-or-break year for climate action, with the window to prevent the worst impacts of climate change—which include ever more frequent more intense droughts, floods and storms—closing rapidly." The problem no longer belongs to a few countries or regions but rather is a worldwide concern measured with increasing temperatures leading to decreased glacier coverage and resulting rise in sea levels.

The good news is, there are small steps that each of us can do that collectively can positively impact climate change.

Making a Positive Impact on Climate Change—Water Usage

Our use of water is one element that impacts climate change. Having access to clean, potable water is critical for not only our health but also for the health of our ecosystem. About 1 out of 10 people on our planet do not have easy access to clean water to drink. As each of us conserves water, we prolong the life span of fresh water from our lakes and rivers and also reduce the impact on sewer systems and drainage in our communities. Additionally, as we conserve water, we also conserve electricity that is used to bring water to and in our homes. So, what can we do to help conserve water?

Example 1 **Brushing Your Teeth (One Person's Contribution)**

Brushing your teeth with the water running continually uses about 4 gal of water. Turning the faucet off when you are not rinsing uses less than one-fourth of a gallon of water. Considering the recommendation to brush your teeth twice a day, how much water would be saved in a week if the faucet was off when not rinsing?

Solution

Leaving the water running continually:

Step 1: Calculate gallons used not with water running continually:

Brushing twice a day for 7 days using 4 gal of water for each brushing

$$(2\text{ times a day})(7\text{ days})(4\text{ gal}) = 56\text{ gal}$$

Step 2: Calculate gallons used turning the faucet off when you are not rinsing:

Brushing twice a day for 7 days using 0.25 gal of water for each brushing

$$(2\text{ times a day})(7\text{ days})(0.25\text{ gal}) = 3.5\text{ gal}$$

Step 3: Calculate savings:

$$\begin{aligned}\text{Savings} &= 56\text{ gal} - 3.5\text{ gal}\\ &= 52.5\text{ gal}\end{aligned}$$

During one week, 52.5 gal of water would be saved if one person turned the faucet off except when rinsing when brushing your teeth.

Example 2 **Brushing Your Teeth (Multiple People's Contribution – Town)**

Using the data in , how much water would be saved in a month if one-fifth of a town's population of 15,000 turned the faucet off when brushing their teeth except when rinsing?

Solution

Step 1: From , we found that 1 person saves 52.5 gal per week.

Step 2: Calculate the population to save water:

$$\text{One-fifth of } 15,000 \text{ people} = 3,000 \text{ people}$$

Step 3: One-fifth of a town's population turning off the faucet when brushing their teeth for a month:

$$(3,000)(52.5 \text{ gal per week})(4 \text{ weeks}) = 630,000 \text{ gal}$$

During one month, 630,000 gallons of water would be saved if one-fifth of a town of 15,000 people turned the faucet off except when rinsing when brushing their teeth.

Example 3 **Brushing Your Teeth (Multiple People's Contribution – State)**

Using the data in , how much water would be saved in a year if one-fourth of the population of the state of Minnesota, which is approximately 5.6 million people, turned the faucet off when brushing their teeth except when rinsing for a year (52 weeks)?

Solution

Step 1: From , we found that 1 person saves 52.5 gal per week.

Step 2: Calculate the population to save water:

$$\text{One-fourth of } 5.6 \text{ million people} = 1.4 \text{ million people}$$

Step 3: One-fourth of a town's population turning off the faucet when brushing their teeth for a month:

$$(1.4 \text{ million})(52.5 \text{ gal per week})(52 \text{ weeks}) = 3,822 \text{ million gal}$$

During one year, 3,822 million gal of water would be saved if one-fourth of the state of Minnesota turned the faucet off except when rinsing when brushing their teeth.

History of Solar Energy

In the mid-1800s, Willoughby Smith discovered photoconductive responsiveness in selenium. Shortly thereafter, William Grylls Adams and Richard Evans Day discovery that selenium can produce electricity if exposed to the sun was a major breakthrough. Less than 10 years later, Charles Fritts invented the first solar cells using selenium. Jumping a mere 100 years later, Bell Labs in the United States produced the first practical photovoltaic cells in the mid-1950s and developed versions used to power satellites in the same decade.

Solar panel use has exploded in recent decades and is now used by residences, organizations, businesses, and government buildings such as the White House, space to power satellites, and various methods of transportation. One reason for the expansion is a continuing drop in cost combined with an increase in performance and durability. In the mid-1950s, the cost of a solar panel was around $300 per watt capability. Twenty years later, the cost was a third of the 1950s' cost. Currently, solar panel cost has dropped to less than $1 per watt while decreasing in size as well as increasing in longevity. The dropping price and improved performance has moved solar to a modest investment that can pay for itself in less than half the time of systems from 15 years ago.

WHO KNEW?

Solar Power's Age

The sun has been harnessed by humans for centuries. The earliest recorded use of tapping the sun's energy for power dates back to the seventh century BC when man focused the sun's rays through a magnifying glass to create fire. Four thousand years later, we find historical record of using mirrors to focus the sun and light torches, often for ceremonial proceedings. Use of the sun to light torches continued through the centuries and has been recorded by various cultures including the Chinese civilization in 20 AD and beyond.

In more recent years, the sun was harnessed to power ovens on ships traversing to oceans in the 1700s. At the same time, the power of the sun was utilized to power steamboats through the 1800s. Mária Telkes, a Hungarian-born American scientist, invented a widely deployed solar seawater distiller used on World War II life rafts. Soon after, she partnered with architect Eleanor Raymond to design the first modern home to be completely heated by solar power. Air warmed on rooftop collectors transferred heat to salts, which stored the heat for later use.

Although solar panels as we know them today are relatively new in history, use of the sun to harness power is much older.

Compute Power Needs for Common Home Devices

A **kilowatt** (kW) is 1,000 watts (W). A **kilowatt-hour** (kWh) is a measurement of energy use, which is the amount of energy used by a 1,000-watt device to run for an hour. Using the definition of a kilowatt-hour, to calculate how long it would take to consume 1 kWh of power, we divide 1,000 by the watts use of a device.

FORMULA

$$1,\frac{000}{\text{watts}} = \text{time needed to use 1 kW}$$

For example, a 75 W bulb would take $1,000 \div 75 = 13.3$ hours to use 1 kW of power.

FORMULA

$$\frac{\text{watts}}{1},000 = \text{kilowatt hours}$$

Example 4 **Calculating the Kilowatt-Hours Needed to Run a Television**

A 48 in plasma television uses about 200 W. How many kilowatt-hours are needed to run the television in a month if the television is one for an average of 2.5 hours a day?

Solution

Step 1: $1,\frac{000}{200\text{ watts}} = 5$ hours to use 1 kW

Step 2: (2.5 hours a day)(30 days) = 75 hours of use

Step 3: $75\ \frac{\text{hour}}{5}$ hours per kW = 15 kW

The television will consume about 15 kW in a month.

Example 5 **Calculating the Cost to Run a Refrigerator**

A medium-sized Energy Star–rated refrigerator uses about 575 W and runs for about 8 hours per day. What is the monthly (30 days) cost of running the refrigerator if the electric rate is 12 cents per kilowatt-hour?

Solution

Step 1: Calculate the watts per day:

$$(575\text{ W})(8\text{ hours}) = 4,600\text{ W per day}$$

Step 2: Calculate the kilowatt-hours.

$$\frac{4,600}{1,000} = 4.6\text{ kWh}$$

Step 3: Calculate the daily cost.

$$(4.6\text{ kWh})(12\text{ cents}) = 55\text{ cents} = \$\,0.55$$

Step 4: Calculate the monthly cost.

$$(\$\,0.55)(30) = \$\,16.50$$

It would cost about \$16.50 to run the refrigerator for a month.

Example 6 **Calculating the Kilowatt-Hours to Run an Oven**

An electric oven is labeled as 4,000 W. How much would it cost to bake a cake for 30 minutes if the electric rate is 14 cents per kilowatt-hour?

Solution

Step 1: Determine the time it takes to use 1 kW of power:

$$1,\frac{000}{4,000\text{ watts}} = 0.25\text{ hours to use 1 kW}$$

For every 15 minutes, the oven uses 1 kW of power.

Step 2: Determine how many kilowatt-hours are needed to bake the cake for 30 minutes:

$$\frac{30\text{ minutes}}{15\text{ minutes per kW}} = 2\text{ kW}$$

Step 3: Calculate the cost of the oven usage:

$$(2\text{ kW})(14\text{ cents per kWh}) = 28\text{ cents}$$

It would cost about 28 cents to bake the cake.

Solar Advantages

There are multiple advantages that solar power can offer us today including reducing greenhouse gas and CO_2 emissions, powering vehicles, reducing water pollution, reducing strain on limited supply of other power options such as fossil fuels. We will look further at reducing greenhouse gas and CO_2 emissions.

Any gas that prevents infrared radiation from escaping Earth's atmosphere is a greenhouse gas. There are 24 currently identified greenhouse gases of which carbon dioxide is one. When measuring the impact of any of the greenhouse gases, the measurements are given in units of carbon dioxide emissions. For this reason, greenhouse gas and carbon dioxide have become interchangeable in discussions.

PEOPLE IN MATHEMATICS

Charles Fritts and Mohammad M. Atalla

Charles Fritts, a New York inventor, is credited with creating the first solar cell, which he installed on a rooftop in New York City in 1884. While the solar cell was not very efficient, having a rate of conversion between 1 to 2%, this was a major step early in solar power energy. Today's solar cells have an efficiency on average of 15 to 20%, which yields a notably higher impact. Nonetheless, the work that Fritts successfully completed marked the start of solar energy through the use of photovoltaic solar panels in the United States.

Mohamed M. Atalla was an Egyptian-born scientist who moved to the United States to complete his studies, and undertook research and development at Bell Laboratories in New Jersey. Many of the early efficiency gains in solar cells were due to his development of processes for using silicon within electronic devices. Atalla's work led to the invention of silicon transistors and microchips (including his own invention of the MOSFET, the most widely used transistor in the world), and quickly increased the efficiency of solar cells.

Example 7 **Calculating the Solar Power for Average Home Use in Kilowatts**

If a home uses approximately 30 kW of electricity per day, what size solar system would be needed to fuel 80% of a home's needs for a month (30 days)?

Solution

$$(30 \text{ kW hours})(30 \text{ days})(0.80) = 720 \text{ kW}$$

A solar system capable of producing 720 kW a month would be needed.

Key Terms

- greenhouse gas
- CO_2 emissions

- watt
- kilowatt (kW)

Key Concepts

- Recognize how water conservation by one person, family, community, or nation can positively impact the world's freshwater supply.
- Recall components from the history of solar use by mankind.
- Calculate electrical demand given watts.
- Recognize advantages of residential solar power.

Formulas

$$1,\frac{000}{\text{watts}} = \text{time needed to use 1 kWh}$$

$$\frac{\text{watts}}{1},000 = \text{kilowatt hours}$$

13.3 Math and Medicine

Figure 9: Shoppers wear masks during the Covid-19 pandemic.

Learning Objectives

After completing this section, you should be able to:

1. Compute the mathematical factors utilized in concentrations/dosages of drugs.
2. Describe the history of validating effectiveness of a new drug.
3. Describe how mathematical modeling is used to track the spread of a virus.

   The pandemic that rocked the world starting in 2020 turned attention to finding a cure for the Covid-19 strain into a world race and dominated conversations from major news channels to households around the globe. News reports decreeing the number of new cases and deaths locally as well as around the world were part of the daily news for over a year and progress on vaccines soon followed. How was a vaccine able to be found so quickly? Is the vaccine safe? Is the vaccine effective? These and other questions have been raised through communities near and far and some remain debatable. However, we can educate ourselves on the foundations of these discussions and be more equipped to analyze new information related to these questions as it becomes available.

Concentrations and Dosages of Drugs

Consider any drug and the recommended dosage varies based on several factors such as age, weight, and degree of illness of a person. Hospitals and medical dispensaries do not stock every possible needed concentration of medicines. Drugs that are delivered in liquid form for intravenous (IV) methods in particular can be easily adjusted to meet the needs of a patient. Whether administering anesthesia prior to an operation or administering a vaccine, calculation of the concentration of a drug is needed to ensure the desired amount of medicine is delivered.

The formula to determine the volume needed of a drug in liquid form is a relatively simple formula. The volume needed is calculated based on the required dosage of the drug with respect to the concentration of the drug. For drugs in liquid form, the concentration is noted as the amount of the drug per the volume of the solution that the drug is suspended in which is commonly measured in g/mL or mg/mL.

Suppose a doctor writes a prescription for 6 mg of a drug, which a nurse calculates when retrieving the needed prescription from their secure pharmaceutical storage space. On the shelves, the drug is available in liquid form as 2 mg per mL. This means that 1 mg of the drug is found in 0.5 mL of the solution. Multiplying 6 mg by 0.5 mL yields 3 mL, which is the volume of the prescription per single dose.

FORMULA

$$\text{Volume needed} = (\text{medicine dosage required})\ (\text{weight of drug by volume})$$

.

A common calculation for the weight of a liquid drug is measured in grams of a drug per 100 mL of solution and is also called the percentage weight by volume measurement and labeled as % w/v or simply w/v.

CHECKPOINT

Note that the units for a desired dose of a drug and the units for a solution containing the drug or pill form of the drug must be the same. If they are not the same, the units must first be converted to be measured in the same units.

Suppose you visit your doctor with symptoms of an upset stomach and unrelenting heartburn. One possible recourse is sodium bicarbonate, which aids in reducing stomach acid.

Example 1 **Calculating the Quantity in a Mixture**

How much sodium bicarbonate is there in a 250 mL solution of 1.58% w/v sodium bicarbonate?

Solution

1.58% $\frac{\text{w}}{\text{v}}$ = 1.58 g sodium bicarbonate in 100 mL. If there is 250 mL of the solution, we have 2.5 times as much sodium bicarbonate as in 100 mL. Thus, we multiply 1.58 by 2.5 to yield 3.95 g sodium bicarbonate in 250 mL solution.

Example 2 **Calculating the Quantity of Pills Needed**

A doctor prescribes 25.5 mg of a drug to take orally per day and pills are available in 8.5 mg. How many pills will be needed each day?

Solution

The prescription and the pills are in the same units which means no conversions are needed. We can divide the units of the drug prescribed by the units in each pill: $\frac{25.5}{8.5} = 3$. So, 3 pills will be needed each day.

Example 3 **Calculating the Drug Dose in Milligrams, Based on Patient Weight**

A patient is prescribed 2 mg/kg of a drug to be delivered intramuscularly, divided into 3 doses per day. If the patient weighs 45 kg, how many milligrams of the drug should be given per dose?

Solution

Step 1: Calculate the total daily dose of the drug based on the patient's weight (measured in kilograms):

$$\left(2\ \frac{\text{mg}}{\text{kg}}\right)(45\ \text{kg}) = 90\ \text{mg}$$

Step 2: Divide the total daily dose by the number of doses per day:

$$90\ \frac{\text{mg}}{3} = 30\ \text{mg}$$

The patient should receive 30 mg of the drug in each dose.

CHECKPOINT

Note that the units for a patient's weight must be compatible with the units used in the medicine measurement. If they are not the same, the units must first be converted to be measured in the same units.

Example 4 **Calculating the Drug Dose in Milliliters, Based on Patient Weight**

A patient is prescribed 2 mg/kg of a drug to be delivered intramuscularly, divided into 3 doses per day. If the drug is available in 20 mg/mL and the patient weighs 60 kg, how many milliliters of the drug should be given per dose?

Solution

Step 1: Calculate the total daily dose of the drug (measured in milligrams) based on the patient's weight (measured in kilograms):

$$\left(2\ \frac{\text{mg}}{\text{kg}}\right)(60\ \text{kg}) = 120\ \text{mg}$$

Step 2: Calculate the volume in each dose:

$$\frac{120\ \text{mg daily total}}{3\ \text{doses a day}} = 40\ \text{mg per dose}$$

Step 3: Calculate the volume based on the strength of the stock:

$$\frac{\text{prescribed dose needed}}{\text{stock dose}} = \text{volume}$$

$$\frac{40\ \text{mg per dose}}{20\ \frac{\text{mg}}{\text{mL}}} = 2\ \text{mL}$$

The patient should receive 2 mL of the stock drug in each dose.

WHO KNEW?

Math Statistics from the CDC

The Centers for Disease Control and Prevention (CDC) states that about half the U.S. population in 2019 used at least one prescription drug each month, and about 25% of people used three or more prescription drugs in a month. The resulting overall collective impact of the pharmaceutical industry in the United States exceeded $1.3 trillion a year prior to the 2020 pandemic.

Validating Effectiveness of a New Vaccine

The process to develop a new vaccine and be able to offer it to the public typically takes 10 to 15 years. In the United States, the system typically involves both public and private participation in a process. During the 1900s, several vaccines were successfully developed, including the following: polio vaccine in the 1950s and chickenpox vaccine in the 1990s. Both of these vaccines took years to be developed, tested, and available to the public. Knowing the typical timeline for a vaccine to move from development to administration, it is not surprising that some people wondered how a vaccine for Covid-19 was released in less than a year's time.

Lesser known is that research on coronavirus vaccines has been in process for approximately 10 years. Back in 2012, concern over the Middle Eastern respiratory syndrome (MERS) broke out and scientists from all over the world began working on researching coronaviruses and how to combat them. It was discovered that the foundation for the virus is a spike protein, which, when delivered as part of a vaccine, causes the human body to generate antibodies and is the platform for coronavirus vaccines.

When the Covid-19 pandemic broke out, Operation Warp Speed, fueled by the U.S. federal government and private sector, poured unprecedented human resources into applying the previous 10 years of research and development into targeting a specific vaccine for the Covid-19 strain.

PEOPLE IN MATHEMATICS

Shibo Jiang

Dr. Shibo Jiang, MD, PhD, is co-director the Center for Vaccine Development at the Texas Children's Hospital and head of a virology group at the New York Blood Center. Together with his colleagues, Jiang has been working on vaccines and treatments for a range of viruses and infections including influenzas, HIV, Sars, HPV and more recently Covid-19. His work has been recognized around the world and is marked with receiving grants amounting to over $20 million from U.S. sources as well as the same from foundations in China, producing patents in the United States and China for his antiviral products to combat world concerns.

Jiang has been a voice for caution in the search for a vaccine for Covid-19, emphasizing the need for caution to ensure safety in the development and deployment of a vaccine. His work and that of his colleagues for over 10 years on other coronaviruses paved the way for the vaccines that have been shared to combat the Covid-19 pandemic.

Mathematical Modeling to Track the Spread of a Vaccine

With a large number of people receiving a Covid-19 vaccine, the concern at this time is how to create an affordable vaccine to reach people all over the world. If a world solution is not found, those without access to a vaccine will serve as incubators to variants that might be resistant to the existing vaccines.

As we work to vaccinate the world, attention continues with tracking the spread of the Covid-19 and its multiple variants. **Mathematical modeling** is the process of creating a representation of the behavior of a system using mathematical language. Digital mathematical modeling plays a key role in analyzing the vast amounts of data reported from a variety of sources such as hospitals and apps on cell phones.

When attempting to represent an observed quantitative data set, mathematical models can aid in finding patterns and concentrations as well as aid in predicting growth or decline of the system. Mathematical models can also be useful to determine strengths and vulnerabilities of a system, which can be helpful in arresting the spread of a virus.

The chapter on Graph Theory explores one such method of mathematical modeling using paths and circuits. Cell phones have been helpful in tracking the spread of the Covid-19 virus using apps regulated by regional government public health authorities to collect data on the network of people exposed to an individual who tests positive for the Covid-19 virus.

PEOPLE IN MATHEMATICS

Gladys West

Figure 10: Gladys West

Dr. Gladys West is a mathematician and hidden figure with a rich résumé of accomplishments spanning Air Force applications and work at NASA. Born in 1930, West rose and excelled both academically and in her professional life at a time when Black women were not embraced in STEM positions. One of her many accomplishments is the Global Positioning System (GPS) used on cell phones for driving directions.

West began work as a human computer, someone who computes mathematical computations by hand. Considering the time and complexity of some calculations, she became involved in programming computers to crunch computations. Eventually, West created a mathematical model of Earth with detail and precision that made GPS possible, which is utilized in an array of devices from satellites to cell phones. The next time you tag a photo or obtain driving directions, you are tapping into the mathematical modeling of Earth that West developed.

Consider the following graph :

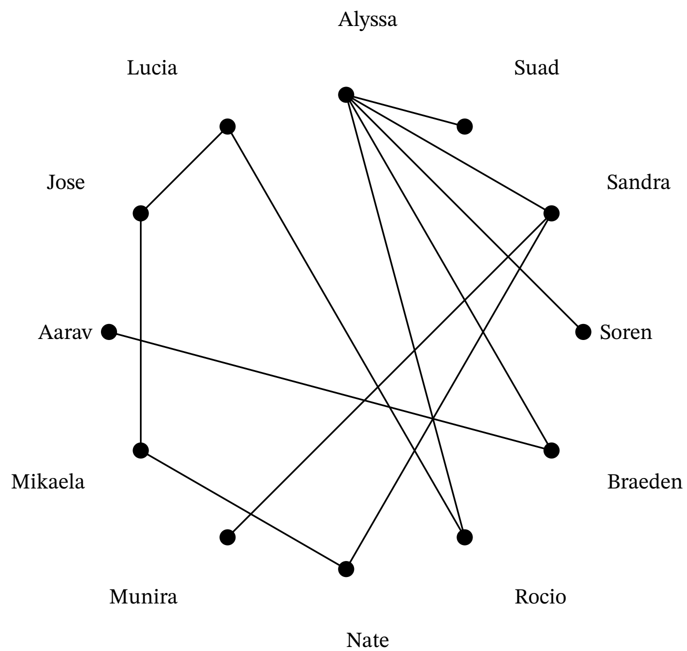

Figure 11: Contact Tracing for Math 111 Section 1

At the center of the graph, we find Alyssa, whom we will consider positive for a virus. Utilizing the technology of phone apps voluntarily installed on each phone of the individuals in the graph, tracking of the spread of the virus among the 6 individuals that Alyssa had direct contact with can be implemented, namely Suad, Rocio, Braeden, Soren, and Sandra.

Let's look at José's exposure risk as it relates to Alyssa. There are multiple paths connecting José with Alyssa. One path includes the following individuals: José to Mikaela to Nate to Sandra to Alyssa. This path contains a length of 4 units, or people, in the contact tracing line. There are 2 more paths connecting José to Alyssa. A second path of the same length consists of José to Lucia to Rocio to Braeden to Alyssa. Path 3 is the shortest and consists of José to Lucia to Rocio to Alyssa. Tracking the spread of positive cases in the line between Alyssa and José aids in monitoring the spread of the infection.

Now consider the complexity of tracking a pandemic across the nation. Graphs such as the one above are not practical to be drawn on paper but can be managed by computer programs capable of computing large volumes of data. In fact, a computer-generated mathematical model of contact tracing would look more like a sphere with paths on the exterior as well as on the interior. Mathematical modeling of contact tracing is complex and feasible through the use of technology.

Example 5 **Using Mathematical Modeling**

For the following exercises, use the sample contact tracing graph to identify paths .

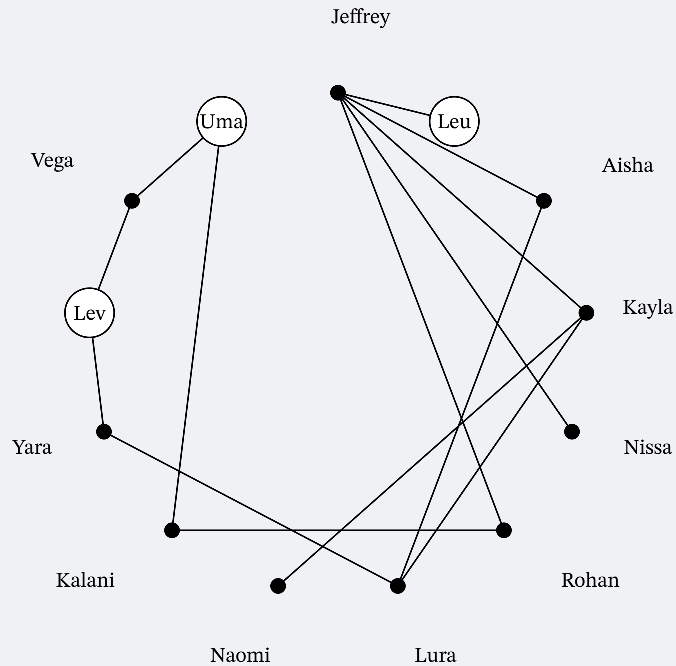

Figure 12: Contact Tracing for ECON 250 Section 1

1. How many people have a path of length 2 from Jeffrey?
2. Find 2 paths between Kayla and Rohan.
3. Find the shortest path between Yara and Kalani. State the length and people in the path.
   Solution
   1. 5 (Lura, Naomi, Kalani, Vega, Yara)
   2. Answers will vary. Two possible answers are as follows:
      1. Kayla, Jeffrey, Rohan
   3. Kayla, Lura, Yara, Lev, Vega, Uma, Kalani, Rohan
   4. Length is 4. People in path = Yara, Lev, Vega, Uma, Kalani

Key Terms

- concentrations/dosages of drugs
- mathematical modeling

Key Concepts

- Compute volumes of prescription drugs in liquid and pill form.
- Validate the effectiveness of a new drug.
- Mathematical modeling can be used to describe and track the spread of a virus.

Formula

$$\text{Volume needed} = (\text{medicine dosage required})\,(\text{weight of drug by volume})$$

.

13.4 Math and Music

Figure 13: Friends sing music together around a campfire.

Learning Objectives

After completing this section, you should be able to:

1. Describe the basics of frequency related to sound.
2. Describe the basics of pitch as it relates to music.
3. Describe and evaluate musical notes, half-steps, whole steps, and octaves.
4. Describe and find frequencies of octaves.

"The world's most famous and popular language is music."

Psy, South Korean singer, rapper, songwriter, and record producer

Imagine a world without music and many of us would struggle to fill the void. Music uplifts, inspires, heals, and generally adds dimension to virtually every aspect of our lives. But what is music? For some it is a song; for others it may be the sounds of birds or the rhythmic sound of drumming or a myriad of other sounds. Whatever you consider music, it is all around us and is an integral part of our lives. "Music can raise someone's mood, get them excited, or make them calm and relaxed. Music also—and this is important—allows us to feel nearly or possibly all emotions that we experience in our lives. The possibilities are endless" (Galindo, 2009).

What music you listen to can impact your mood and emotions. In similar fashion, the music we choose can often tell those around us something about our current moods and emotions. Consider the music you may have been listening to as you today or even as you are reading this text. What cues to your mood do your music selections share? Albert Einstein is quoted as saying, "If I were not a physicist, I would probably be a musician. I often think in music.

I live my daydreams in music. I see my life in terms of music." What clues do your recent music choices say about your mood or how your day is going?

Basics of Frequency as It Relates to Sound

Every sound is created by an object vibrating and these vibrations travel in waves that are captured by our ears. Some vibrations we may be able to see, such as a plucked guitar string moving, whereas other vibrations we may not be able to see, such as the sound created when we hold our breath when accidentally dropping our cell phone on a hard floor. We don't see the vibrations of our cell phone hitting the floor; however, any audible sound created in the fall is the result of vibrations in the form of sound waves, which can be pictured similarly to waves moving through the ocean.

The waves of sounds each have a **frequency**, or rate of vibration of sound waves, that measures the number of waves completed in a single second and are measured in **hertz** (Hz; one Hz is one cycle per second). Louder sounds have stronger vibrations or are created closer to our ear. The further an ear is from the source of the sound, the quieter the sound will appear.

Sounds range in frequency from 16 Hz to ultrasonic values, with humans able to hear sounds in a frequency range of about 20 Hz to 20,000 Hz. Adults lose the ability to hear the upper end of the range and typically top out in the ability to hear in a frequency of 15,000–17,000 Hz. Sounds with a frequency above 17,000 Hz are less likely to be heard by adults while still being audible to children.

While frequency plays a key role in audible sounds, so too does the sound level, which can be measured in **decibels** (dB), which are the units of measure for the intensity of a sound or the degree of loudness. As a sound level increases, the decibel level increases.

A person with average hearing can hear sounds down to 0 dB. Those with exceptionally good hearing can hear even quieter sounds, down to approximately −5 dB. The following table includes sample sounds with their related decibel values.

Sound	Decibels (dB)
Firecrackers	140
Take-off of military jet from aircraft carrier	130
Clap of thunder	120
Auto horn standing next to the vehicle	110
Outboard motor	100
Motorcycle	90
Noise inside a car in city traffic	80
Typical washing machine	70
Public conversation, such as at a restaurant	60
Private conversation	50
Hum of a computer with fan blowing	40
Quiet whisper	30
Swishing leaves	20
Regular breathing	10
Lowest typical sound audible by teenagers	0

Example 1 **Selecting Decibel Value of Sounds**

Select the most representative decibel value for each of the following sounds:

1. car wash: 25 dB, 55 dB, 85 dB
2. vacuum cleaner: 15 dB, 70 dB, 90 dB
3. ship's engine room: 30 dB, 65 dB, 95 dB
4. approaching subway car: 70 dB, 100 dB, 120 dB

Solution

1. 85 dB
2. 70 dB
3. 95 dB
4. 100 dB

Basics of Pitch

When considering the various sound levels the human ear can hear, the ear perceives sound both from the frequency level and the pitch of a sound. The quality of the sound is referred to as

pitch, the tonal quality of a sound and how high or low the tone. Sounds with a high frequency have a high pitch, such as 900 Hz, and sounds with a low pitch have a low frequency, such as 50 Hz.

Let's take a look at frequency and pitch using a string instrument such as a guitar or piano. When a string is plucked on a guitar or a key is played on a piano, the related string vibrates at a frequency that is related to the length and thickness of the string. The frequency is measurable and has a singular value. The pitch of the note played is open for interpretation, as the pitch is a function of personal opinion.

WHO KNEW?

Graphing Calculators and Music

It may be interesting to note that a TI-84 and TI-Nspire graphing calculators can be utilized to tune a musical instrument by measuring the frequency of a note using a small plug-in accessory that captures the sound waves from a note and displays the corresponding frequency. Using the displayed frequency, an instrumentalist can then make the needed adjustment to perfectly tune an instrument. This method of tuning an instrument can be helpful whether a novice player or a seasoned instrumentalist because the instrument can be tuned precisely to the correct frequency.

Note Values, Half-Steps, Whole Steps, and Octaves

"There are not more than five musical notes, yet the combinations of these five give rise to more melodies than can ever be heard."

Sun Tzu, Chinese strategist

Figure 14: Piano Keys and Notes

Moving our exploration to note values, the frequency of all notes is well defined by a specific and unique frequency for each note that is measurable. We will explore keys on a keyboard to discuss notes that have the same relationships with any instrument or musical piece.

Let's look at . The white keys are labeled with the letters A–G and the photo begins with middle C, which can be found in the middle of a keyboard. This labeling of the keys repeats across an entire keyboard and keys to the right have a higher pitch and frequency than keys to the left. Each of the keys correlates to a musical note.

Movement up or down between any two consecutive keys (black and white) or notes constitutes a **half-step**. Movement of one half-step sometimes involves a **sharp** (#) or a **flat** (♭) symbol. For example, D# is one half-step above D and D♭ is one half-step below D. Note that this is not always true as one half-step above B is C, and one half-step below F is E. In similar fashion, a **whole step** is movement up or down between any two half-steps on a keyboard.

Example 2 **Identifying Half-Steps**

Name which keys are one half-step up and one half-step down from the following:

1. D
2. E
3. G#

Solution

1. up D#, down D♭
2. up F, down E♭
3. up A, down G

Example 3 **Identifying Whole Steps**

Name which keys are one whole step up and one whole step down from the following:

1. F#
2. E
3. A♭

Solution

1. Up G#, down E
2. Up F#, down D
3. Up B♭, down G♭

You may have noticed that there are eight letters of the alphabet used to label notes. Selecting any one note and counting up 12 half-steps you will find that the numbering for notes begins at the same value as you started from. This collection of 12 consecutive half-notes is called an **octave** and is a basic foundational component in music theory.

Example 4 **Listing All Notes in an Octave**

List the 12 notes forming an octave, beginning with the note C.

Solution

C, $C^{\#}$, D, $D^{\#}$, E, F, $F^{\#}$, G, $G^{\#}$, A, $A^{\#}$, B

Frequencies of Octaves

Notes that are one octave apart have the same name and are related in frequency values. Given the frequency of any note, the frequency of same note one octave higher is doubled and this pattern continues as you move up and down the notes on a keyboard or any other musical instrument. Song writers and singers use this knowledge to change the pitch of a note up or down to align with a person's vocal range. Regardless of which C is played or sung, the pitch is the same and the frequency is related by a power or two.

Labeled keys on a keyboard are numbered for ease in identification. For example, middle C is labeled as C_4 on a full keyboard as it is the fourth C from the left in a set of eight notes. The frequency of C_4 is 262 Hz, rounded to the nearest whole number.

Example 5 **Calculating the Frequency Values of Octaves**

Given that the frequency of C_4 is 262 Hz, find the approximate frequency of C_6.

Solution

The frequency of each consecutive higher octave doubles. Given that the frequency of C_4 is 262 Hz, the frequency of C_5 is found by doubling the frequency of C_4, which is 524 Hz. In similar fashion, the frequency of C_6 is found by doubling the frequency of C_5, which yields 1,048 Hz.

PEOPLE IN MATHEMATICS

David Cope

David Cope has a somewhat eclectic list of job titles ranging from author and music professor to scientist and artificial intelligence researcher. Cope combined his interests when he developed software that can analyze a piece of music and create a new and unique musical piece in the same style as the original. Some of his well-known products have been based off of the classical music of Mozart, creating what has been called Mozart's "42nd Symphony," as well as other genres including opera and a range of current music styles. Cope has also composed original musical pieces in collaboration with a computer.

We have explored some basics components of frequency, pitch, note relationships, and octaves, which are building blocks of music. It may be exciting to learn that the mathematical relationships found in music are vast and grow in complexity beyond the math commonly studied in high school.

WHO KNEW?

Spotify Royalty Payments

> Streaming services have grown exponentially in popularity thanks in large part to customized music listening through cell phone use and devices such as Google as well as Amazon Echo and Alexa devices for home and vehicles, adding to ways that artists are paid royalties. Spotify, which was launched in 2008, typically plays artists $0.06 per time a song is streamed, with some artists receiving up to $0.84 per play amounting to over $9 billion in revenue for Spotify in 2020. Since 2014, Spotify's revenue has grown over a billion dollars a year, with roughly half of their revenue being paid out in royalties, which was good news for artists during the Covid-19 pandemic when in-person concerts and shopping were hindered.

Key Terms

- frequency
- pitch
- Hertz
- decibel
- half-step
- whole step
- flat
- sharp
- octave

Key Concepts

- As frequency of a sound increases, the pitch of the sound increases.
- Hertz is a unit of measurement for frequency.
- Decibel is a unit of measurement for the intensity of sound.
- Half-steps and whole-steps describe one type of movement between two notes on a keyboard.
- Octaves are a collection of any 12 consecutive notes, which is a foundation in music theory.

13.5 Math and Sports

Figure 15: Fans support their team by attending games and wearing team gear.

Learning Objectives

After completing this section, you should be able to:

1. Describe why data analytics (statistics) is crucial to advance a team's success.
2. Describe single round-robin method of tournaments.
3. Describe single-elimination method of tournaments.
4. Explore math in baseball, fantasy football, hockey, and soccer (projects at the end of the section).

   Sports are big business and entertainment around the world. In the United States alone, the revenue from professional sports is projected to bring in over $77 billion, which includes admission ticket costs, merchandise, media coverage access rights, and advertising. So, whether or not you enjoy watching professional sports, you probably know someone who does. Some celebrities compete to be part of half-time shows and large companies vie for commercial spots that are costly but reach a staggering number of viewers, some who only watch the half-time shows and advertisements.

Data Analytics (Statistics) Is Crucial to Advance a Team's Success

Analyzing the vast data that today's world has amassed to find patterns and to make predictions for future results has created a degree field for **data analytics** at many colleges, which is in high demand in places that might surprise you. One such place is in sports, where being able to analyze the available data on your team's players, potential recruits, opposing team strategies, and opposing players can be paramount to your team's success.

Hollywood turned the notion of using data analytics into a major motion picture back in 2011 with the release of *Moneyball,* starring Brad Pitt, which grossed over $110 million. The critically acclaimed movie, based on a true story as shared in a book by Michael Lewis, follows the story of a general manager for the Oakland Athletics who used data analytics to take a team comprised of relatively unheard of players to ultimately win the American League West title in a year's time. The win caught the eye of other team managers and owners, which started an avalanche of other teams digging into the data of players and teams.

In today's world of sports, a team has multiple positions utilizing data analytics from road scouts who evaluate a potential recruit's skills and potential to the ultimate position of general manager who is typically the highest-paid (non-player) employee with the exception of the coaches. Being able to understand and evaluate the available data is big business and is a highly sought after skill set. In college and professional sports, it is no longer sufficient to have a strong playbook and great players. The science to winning is in understanding the math of the data and using it to propel your team to excelling.

Single Round-Robin Tournaments

8 Team round robin

Team 1		Team 2		Team 3		Team 4	
W	L	W	L	W	L	W	L

Team 5		Team 6		Team 7		Team 8	
W	L	W	L	W	L	W	L

Round 1	Round 2	Round 3	Round 4	Round 5	Round 6	Round 7
2 vs 1	3 vs 4	6 vs 2	7 vs 5	1 vs 3	4 vs 6	7 vs 3
3 vs 8	1 vs 7	7 vs 8	8 vs 4	4 vs 2	8 vs 1	8 vs 2
4 vs 7	8 vs 6	4 vs 1	2 vs 3	5 vs 8	2 vs 7	1 vs 5
5 vs 6	2 vs 5	5 vs 3	6 vs 1	6 vs 7	3 vs 6	6 vs 4

Figure 16: Single-Round Robin Tournament

A common tournament style is single round-robin tournaments , where each team or opponent plays every other team or opponent, and the champion is determined by the team that wins the most games. Ties are possible and are resolved based on league rules.

An advantage of the round-robin tournament style is that no one team has the advantage of **seeding**, which eliminates some teams from playing against each other based on rank of their prior performance. Rather, each team plays every other team, providing equal opportunity to triumph over each team. In this sense, round-robin tournaments are deemed the fairest tournament style.

One hindrance to employing a round-robin-style tournament is the potential for the number of games involved in tournament play to determine a winner. Determining the number of games can be found easily using a formula which, as we will see, can quickly grow in the number of games required for a single round-robin tournament.

FORMULA

The number of games in a single round-robin tournament with n teams is $n\frac{n-1}{2}$.

Example 1 **Calculating the Number of Games in Single Round-Robin Tournaments**

Find the number of games in a single round-robin tournament for each of the following numbers of teams:

1. 4 teams
2. 8 teams
3. 20 teams

Solution

1. Using the formula with 4 teams yields $4\frac{4-1}{2} = 6$ tournament games.
2. Using the formula with 8 teams yields $8\frac{8-1}{2} = 28$ tournament games.
3. Using the formula with 20 teams yields $20\frac{20-1}{2} = 190$ tournament games.

As the examples show, single round-robin tournament play can quickly grow in the number of games required to determine a champion. As such, some tournaments elect to employ variations of single round-robin tournament play as well as other tournament styles such as elimination tournaments.

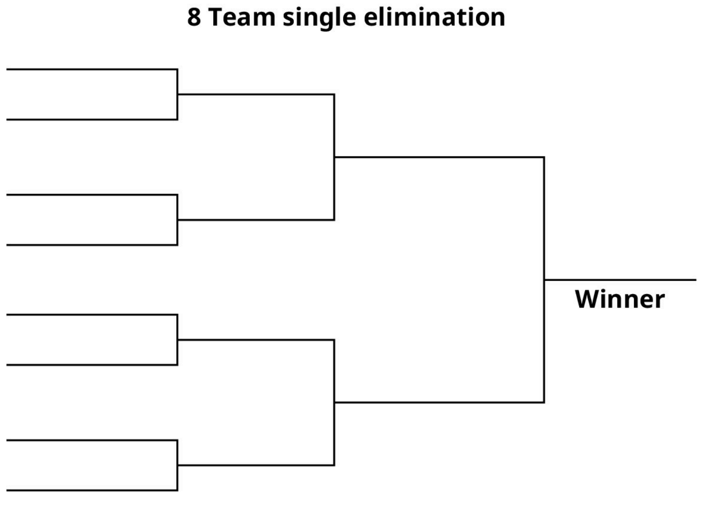

Figure 17: Single-Elimination Tournament

Single-Elimination Tournaments

When desiring a more efficient tournament style to determine a champion, one option is single-elimination tournaments , where teams are paired up and the winner advances to the next round of play. The losing team is defeated from tournament play and does not advance in the tournament, although some leagues offer consolation matches.

FORMULA

The number of games in a single-elimination tournament with n teams is $(n - 1)$.

Example 2 **Calculating the Number of Games in Single-Elimination Tournaments**

Find the number of games in a single-elimination tournament for each of the following numbers of teams:

1. 4 teams
2. 8 teams
3. 20 teams

Solution

1. Using the formula with 4 teams yields $(4 - 1) = 3$ tournament games.
2. Using the formula with 8 teams yields $(8 - 1) = 7$ tournament games.
3. Using the formula with 20 teams yields $(20 - 1) = 19$ tournament games.

A single-elimination tournament offers an advantage over single round-robin tournament style of play in the number of games needed to complete the tournament. As you can see, in comparing the number of games in a single round-robin tournament in Example 1 with the number of games in single-elimination tournament as shown in Example 2, the number of games required for single round-robin can quickly become unmanageable to schedule.

There are modifications to both the round-robin and elimination tournament styles such as double round-robin and double-elimination tournaments. Next time you observe a college or professional sporting event, see if you can determine the tournament style of play.

WHO KNEW?

Sports Popularity Shifts

Sport has been a popular entertainment venue for hundreds of years and the popularity of various sports shifts over time as well as in different regions of the world today. In today's world, the most popular sport is, overwhelmingly, soccer, with over 4 billion fans followed by cricket with 2.5 billion fans. It may surprise you to learn that American football doesn't rank in the top 10 most popular sports in the world today, yet table tennis ranks in sixth place and golf fills the last spot in the top 10 world sports.

In the early 1930s, baseball ranked a close second, with basketball virtually tying for third place. By the mid-1940s, hockey slightly led in first place over basketball. Ten years later, hockey remained in the number one sport, but cricket pushed basketball to third place. In the 1960s, soccer dominated in popularity. Over the next 30 years, basketball dropped in popularity and there was much movement in the popularity of sports. By the late 1990s, soccer swept to first place, where it has since continued to grow in popularity.

Key Terms

- data analytics
- seeding

Key Concepts

- Describe how a round-robin tournament is organized.
- Compute the number of games played in a round-robin tournament.
- Describe how a single-elimination tournament is organized.
- Compute the number of games played in a single-elimination tournament.

Formulas

Number of games in a single round-robin tournament with n teams is $n\frac{n-1}{2}$.

Number of games in a single-elimination tournament with n teams is $(n-1)$.

project

Lucas Sequence and Fibonacci Sequence

The Lucas numbers bear some similarity to the Fibonacci numbers and exhibit a stronger link to the golden ratio. Edouard Lucas is credited with naming the Fibonacci numbers and the Lucas numbers were so named in his honor. The Lucas numbers play a role in finding prime numbers that are utilized in encrypting data for actions such as using your debit card to obtain money at a cash machine or when making a credit card purchase for point of sale as well as when shopping online.

Complete the following questions to explore numbers in the Lucas sequence as well as their relationships to the numbers in the Fibonacci sequence.

1. Conduct an Internet search to find out what a Lucas number is and how the Lucas numbers are related to the Fibonacci numbers.
2. What are the first two numbers in the Lucas sequence?
3. Describe how the next number in the Lucas sequence is determined and compare this to how the next number is determined in the Fibonacci sequence.
4. Complete the following table listing the first 10 terms in the Fibonacci and Lucas sequence:

Term	0	1	2	3	4	5	6	7	8	9
Fibonacci Numbers	0	1	1	2		5		13		34
Lucas Numbers			3		7		18		47	

5. Interestingly, many patterns can be found in looking at the relationships in the Fibonacci and Lucas numbers. Look closely at the chart in question 3 to discover one such pattern. Observe the Fibonacci numbers in the third and fifth terms and compare with the Lucas number in the fourth term of the sequence. Describe the pattern found. Does this pattern continue in the table?

6. Research the Fibonacci or the Lucas numbers to find an application in our world distinct from what has been shared in this project and section. Write a paragraph sharing the findings of your research.

Solar Array for a Residence

One of the first steps in adding solar to a residence is determining the size of a system to achieve the desired output. In this project, we will explore the solar needs of a residence and estimate needs of a solar array to supply electrical output to meet various percentages of electrical need.

Step 1: Obtain an electric bill from your apartment/home. Find the average monthly or yearly usage if listed or call the electric company to inquire. If an electric bill is not available, use the Internet to find an average monthly or yearly electric usage for your area.

Step 2: Determine a daily and hourly usage. Divide the average monthly usage by 30 or the yearly average by 365. Divide again by 24 to calculate an average hourly electric usage, which will yield the average kilowatt-hours for how much electrical power your is being utilized in an hour.

Step 3: Multiply your average hourly use (kilowatts) by 1,000 to convert to watts.

Step 4: Use the Internet to determine the average daily peak hours of sunlight where you live.

Step 5: Divide your average hourly watts (Step 3) by the average daily peak hours (Step 4) to calculate the average energy needed for a solar array to produce every hour.

Step 6: Determine the average energy needed in a solar array per hour to meet each of the following:

1. 100% coverage of average energy (Step 5)
2. 80% coverage of average energy
3. 50% coverage of average energy

   Step 7: Using the values computed in Step 6, compute the residential savings based on an average cost of 12 cents per watt.

Vaccine Validation

Validation of vaccines is a topic that exploded in the news when the Covid-19 pandemic spread across the world. As governments and organizations looked for a vaccine to curb the spread and minimize the severity of infection, concern was expressed by some for what appeared to be a quick discovery for a Covid-19 vaccine.

Conduct an Internet search to explore the following questions. Pay special attention to the sources you select to ensure that they are credible sources.

1. Research the term *efficacy rates*. Express what efficacy means in your own words.
2. Using a minimum of two sources, compose a well-developed paragraph describing what validation of a vaccine means.
3. Using a minimum of two sources, compose a well-developed paragraph sharing the steps in validating a vaccine.

4. Using a minimum of two sources, compose a well-developed paragraph describing how a vaccine is determined to be validated.
5. Using your research for Questions 1–3, write a summary paragraph sharing your reflections on the validation of the Covid-19 vaccine or on validating a vaccine in general. What key components did you learn? What would you like to learn more about related to validating a vaccine?

Frequency and Ultrasonic Sounds

Ultrasonic sounds have been utilized for a variety of reasons, from purportedly repelling rodents and other animals as well as a variety of other applications. Using an Internet search, complete the following questions to explore some of these applications and examine the validity of various claims.

Repelling Insects, Rodents, and Small Animals Some radio stations purport to play a high pitch sound dually with their music to aid in deterring insects and other annoying bugs to aid in providing a bug-reduced listening environment. To deter small rodents, some products claim to emit ultrasonic sounds that drive away mice and other similar pests.

1. Research the science behind ultrasonic pest controls, paying attention to the source of the information that you find. Compare the information found on advertisements, reviews, and scientific articles.
2. What frequency ranges do ultrasonic pest deterrent devices utilize and how do these frequencies compare to the audible range that humans can hear?
3. Write a short paragraph comparing the claims in the advertisements with independent reviews and scientific articles.
4. What do you conclude about ultrasonic pest controls and why?

   Disbursing Teenagers Some business owners and communities have turned to products such as the "mosquito" sonic deterrent device to discourage groups of teenagers from loitering around storefronts and community landmarks, citing a public nuisance issue and public safety concerns.

   1. Research the science behind ultrasonic deterrent devices as they apply to dispensing teenagers. Compare the information found on advertisements, reviews, and scientific articles.
   2. What frequency ranges do ultrasonic teenager deterrent devices utilize and how do these frequencies compare to the audible range that adults can hear?
5. Write a short paragraph comparing the claims in advertisements with independent reviews and scientific articles.
6. What are some of the ethical debates surrounding the use of ultrasonic teenager deterrent devices?
7. What do you conclude about business owners or communities using ultrasonic teenager deterrent devices and why?

Jewelry Cleaner Use of pastes and liquid chemicals to clean jewelry can be harsh on stones as well as metals. So how can we safely obtain the sparkling clean look at home that jewelry stores provide? Some would say the answer is to use an ultrasonic jewelry cleaner, but do these really work?

1. Research the science behind ultrasonic jewelry cleaners, including the phrase "cavitation process" in your search. Compare the information found on advertisements, reviews, and scientific articles.
2. Write a short paragraph detailing how ultrasonic jewelry cleaners work and what role the cavitation process plays in the claims for ultrasonic cleaning.

8. Do ultrasonic jewelry cleaners utilize low or high frequencies? How do these frequencies compare to the audible range that humans can hear?
9. Write a short paragraph comparing the claims in the advertisements with independent reviews and scientific articles.
10. What do you conclude about ultrasonic jewelry cleaners and why?

Specialized Ringtones As the use of cell phones has become commonplace and families grow towards each member having their own cell phone, specialized ringtones have become popular and can aid in identifying who is calling just by the ringtone.

Ever hear of ringtones that can be heard by teens but often not their teachers? The banning of cell phone use by K–12 students during class time as been implemented across a wide array of schools and some students have purportedly found ways to get around teachers hearing a cell phone ring through the use of ultrasonic ringtones.

1. Research the science behind ultrasonic ringtones. Compare the information found on advertisements, reviews, and scientific articles.
2. Do ultrasonic ringtones utilize low or high frequencies?

11. Write a short paragraph detailing how ultrasonic ringtones work. How do these frequencies compare to the audible range that adults can hear? Include ages that are purported to hear and not hear the ringtones as well as the frequency ranges utilized.
12. Write a short paragraph comparing the claims in the advertisements with independent reviews and scientific articles.
13. What do you conclude about ultrasonic ringtones and why?

Streaming Services and Math

With the ability to stream music virtually anywhere you are, it is not surprising that Google Play Music, Apple Music, and a slew of other companies such as Spotify, Amazon Music, YouTube Music, Sound Cloud, Pandora, Deezer Music, Tidal, Napster, and Bandcamp have invested heavily to bring streaming service to users worldwide. Streaming services have expanded to offer virtually every genre of music with vast libraries to meet diverse user requests.

Considering all of the choices available for streaming music, there is a wide array of options for subscribing. Conduct an Internet research to review your current streaming choices, if any, and evaluate competitors' products.

1. In this project, you will explore options for music streaming service subscriptions. Select a minimum of five streaming services listed above that you are not currently utilizing and determine the below components. Format your findings in an easy-to-read format such as a table similar to the one shown below.
2. Monthly subscription cost
3. Available features

Available Features	Service Choice 1:	Service Choice 2:	Service Choice 3:	Service Choice 4:	Service Choice 5:
Able to stream unlimited music					
Able to purchase individual songs					
Able to purchase individual whole albums					
Ability to create personalized play lists					
Ability to select specific songs to play					
Ability to listen					
Other =					

4. Premium cost per month, if available = ______
   1. Feature(s) offered with premium monthly subscription = ______

5. What is (are) your current music streaming services, if any?
    1. What is your current monthly service charge(s)?
6. What features does your current streaming service offer?
7. Write a paragraph summarizing your findings and include if your current streaming service(s) meets your needs or if research for this project has you considering changing streaming services. Support your rationale.

Math and Baseball

Baseball is known to have one of the largest pools of statistics related to the game and its players. Managers, coaches, and pitchers study the statistics of the players on opposing teams to give their team an edge by knowing what pitches to throw for the best probability to be missed by a batter. In similar fashion, batters study pitchers' statistics to learn a pitcher's strength and how to predict what a pitcher will throw and how to best hit against a pitcher.

The three primary baseball statistics are batting average, home runs, and runs batted in (RBIs), which are the components of the title of Triple Crown winner that is awarded to players who dominate in these three areas. However, there is a wealth of other statistics to evaluate when studying the performance of a player.

Conduct an Internet search to research statistics and how they are calculated in the following categories:

Batting Statistics

1. There are about 30 batting statistics. Select a minimum of 10 batting statistics. Compose an organized list including the name of the statistic, abbreviation, explanation of what it represents, as well as how it is calculated.

As an example: AB/HR represents *at bats per home run* and is calculated by the number of times a player is at bat divided by home runs.

Pitching Statistics

1. There are about 40 pitching statistics. Select a minimum of 10 pitching statistics. Compose an organized list including the name of the statistic, abbreviation, explanation of what it represents, as well as how it is calculated.

As an example: K/9 represents *strikeouts per nine innings* and is calculated by the number of strikeouts times nine divided by the number of innings pitched.

Fielding Statistics

1. There are around 10 fielding statistics. Select a minimum of five fielding statistics. Compose an organized list including the name of the statistic, abbreviation, explanation of what it represents, as well as how it is calculated.

As an example: FP represents *fielding percentage* is calculated by the number of total plays divided by the number of total chances.

Overall

1. Select a player from recent years to evaluate. The player can be one that you have followed, one from a favorite team, or any current player. Find the statistics shared in Questions 1–3 to use in evaluating the potential strengths and weaknesses of the player. Write a short paragraph analyzing your selected player, supported by the statistics from the answers to Questions 1–3.

Math and Fantasy Football

Fantasy football offers spectators an added dimension to football season with a competitive math-based game where the active components are real-life players in the current season. For clarity in this exercise, the actual fantasy football players will be denoted as FFP and actual professional team members will be denoted as players.

While some fantasy football leagues have slightly different setups or scoring systems, most share some common elements.

Often using a lottery system to determine who picks first, second, and so on, FFPs select 15 current players to comprise their personal fantasy football team. The players selected can be from any professional teams and a FFP can utilize any team recognized by their league. FFPs can elect to keep the same players on their team for the whole league play or trade for any player not selected by another FFP in their league.

At the start of each week during football season, each fantasy football player selects their roster of actual players to comprise their roster of starting players. Typically, a starting roster consists of the following players:

Number to Select	Position Abbreviation	Position Title
1	QB	Quarterback
1	K	Kicker
1	TE	Tight End
2	RB	Running Back
1	D/ST	Defense
2	WR	Wide receiver
1	RB or WR	Flex

As actual professional games are played, points are tallied based on your league's scoring system. The points the team members on your starting roster make during the week are computed and whichever FFP has the highest score for the week wins that week.

The FFPs with the best records of wins versus losses enters fantasy football playoffs to determine the ultimate league champion and collects the pot.

The above overview of fantasy football describes the basic game play. The fun comes in understanding and analyzing the math behind the scoring.

1. Talk to people you know who have played fantasy football in the past and interview them. Inquire about how they select players and how league(s) they have played in have computed the weekly scoring.
2. Was a league manager recruited to manage the record keeping of scores?
3. Was an online scoring system utilized or did the fantasy football use a streamlined or specialized scoring system?
4. What strategies were used to select the team of 15 players and who would be on the weekly starting roster?
5. Were any online resources utilized to aid in choosing which players to select?
6. How was math utilized to select team members or who to place as a starter each week? Be as specific as possible, citing examples when available.
7. Conduct an Internet research to find two different resources that a FFP could utilize when selecting players for their team or for their starting roster. Describe the math involved in the resources.
8. Conduct an Internet research to learn more about the scoring utilized by fantasy football leagues. Write a short paragraph sharing how the use of math and knowledge of football statistics is used and how they are calculated to aid a FFP in selecting their team and starting roster.

Math and Hockey

Hockey is full of math from obvious components such as scoring and statistics to the shape of the rink and the angles involved in puck movement.

Collegiate and professional hockey games are 60 minutes long and are divided into three periods of 20 (60/3) minutes each. At any one time, there are five players and one goalie on the ice for each team. If a player is called on a penalty and is placed in the penalty box, that team now has four players, which is 20% less players on the ice competing against five opponents. In some instances, a team may have two players in the penalty box at one time, resulting in three players, or 40% less players on the ice compared to a full team. Being one player down for a 2-minute penalty or potentially 5 minutes for a major penalty leads to an imbalance on the ice and calls for a quick change of offense and defense strategy.

Rink Composition—North American An ice rink is comprised of various geometrical shapes, each with precise dimensions.

1. Research the dimensions of a hockey rink and draw an accurate scale model on graph paper. Be sure to include the scale for your model and indicate all units.
2. List the shapes and the numbers of each kind of hockey rink. You should find four different shapes, some with multiple dimensions.
3. Describe the shape and dimensions of a hockey puck using geometric vocabulary.

   Statistics Using an Internet search, select two top hockey players from the same league to answer the following questions:

1. Write a paragraph sharing a minimum of five statistics for each player you have selected. Describe how each of the statistics are calculated and what each statistic means.

4. Write a paragraph comparing the two players and determine who you believe is the better player. Support your choice.

   Scoring The basics of scoring in ice hockey is simple, the team with the most goals is the winner. But, how to score the most goals involves much math!

   1. Research one of the following components involved in hockey puck movement on the ice and write a paragraph summarizing your findings. Be specific and detailed in your summary.
   2. Angles

5. Velocity vectors
6. Angle of incident and angle of return
7. Speed and acceleration (player as well as puck movement)

Math and Soccer

As the world's most popular sport, you'll be excited to confirm that soccer is full of mathematics ranging from scoring and statistics to footwork, angles, and field shape.

Soccer requires understanding of mathematical concepts and equations as well as skill, fitness, and game knowledge. One such example is angles, which you all will remember from your geometry class. While players are not carrying protractors and measuring angles during play, mental calculation of angles is a constant in any successful player's thinking. A goalie is not physically able to cover the entire open net region and a player must calculate an angle to kick the ball consistent with the net opening while predicting the ability of the goalie to stop the ball from entering the net.

1. Research angles as they apply to soccer play. Provide two examples of different plays indicating angle of a player's body, angle of foot striking the ball, and angle to the net. Include relevant dimensions. Adding a diagram that may aid in clarity is an option.

2. Draw a scale model of a soccer field including dimensions with labels.

   Using an Internet search, select two top soccer players from the same league to answer the following questions:

   1. Write a paragraph sharing a minimum of five statistics for each player you have selected. Describe how each of the statistics are calculated and what each statistic means.

3. Write a paragraph comparing the two players and determine who you believe is the better player. Support your choice.