Chapter 6: Money Management

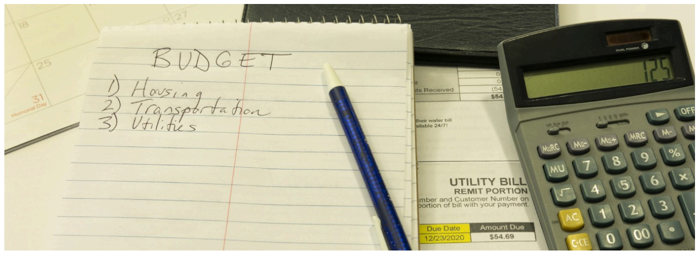


Figure 1: Financial health helps you realize your goals.

The topic of money management is a broad and sometimes complex one. Ultimately, personal money management involves managing both our debt and also our savings and investments.

In 2021, the average American had consumer debt balance of $96,371. Nearly $100,000 per person. And less than 25% of Americans are debt free. Consumer debt can include mortgages, credit cards, as well as student loans. A key question all consumers should consider is how to manage debt and not become overburdened by it. The first step is to create a budget, which puts earnings into perspective, indicating what we can, and cannot, afford. A budget also entails setting aside certain funds for savings and investment, which help us achieve our short- and long-term goals.

Creating a budget requires an understanding of how money—debt and savings—works. Initially, percentages and interest need to be understood. They drive most of what happens with debt and savings. With that understanding, discussions of buying a house, a car, or incurring credit card debt can be addressed from a financial perspective. All the while, retirement is waiting. Preparing for retirement involves saving and saving earlier rather than later. The power of compound interest is on full display when saving early.

This chapter covers some of the basics of money management: percentages, interest, budgeting, debt (student loans, mortgage, car, credit cards), savings, investments, and taxes.

6.1 Understanding Percent

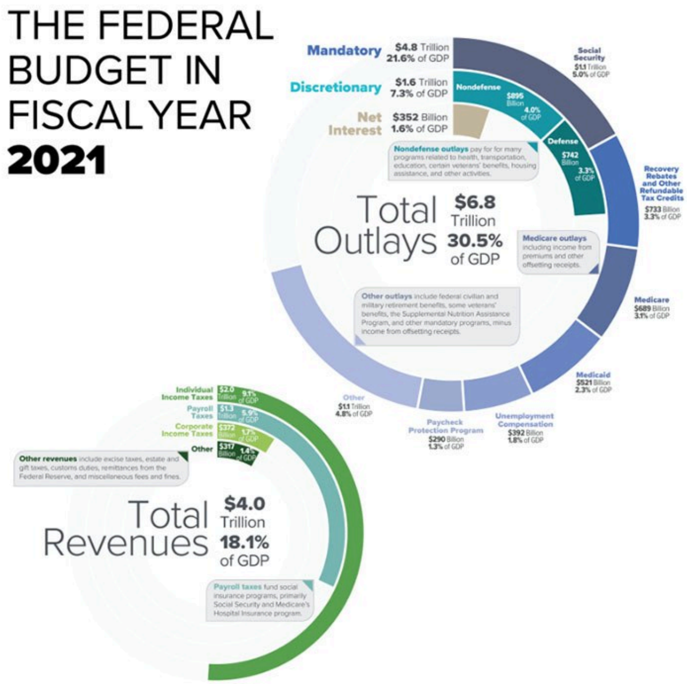


Figure 2: The federal budget describes how money is spent and how money is earned.

Learning Objectives

After completing this section, you should be able to:

1. Define and calculate percent.
2. Convert between percent, decimal, and fractional values.
3. Calculate the total, percent, or part.
4. Solve application problems involving percents.

In 2020, the U.S. federal government budgeted $3.5 billion for the National Park Service, which appears to be a very large number (and is!) and a large portion of the total federal budget. However, the total outlays from the U.S. federal government in 2020 was $6.6 trillion. So, the amount budgeted for the National Park Service was less than one-tenth of 1 percent, or 1/10%, of the total outlays. This **percent** describes a specific number. Understanding that ratio puts the $3.5 billion budgeted to the National Park Service in perspective.

This chapter focuses on percent as a primary tool for understanding money management. The interest paid on debt, the interest earned through investments, and even taxes are entirely determined using percent. This section introduces the basics of working with this invaluable tool.

Define and Calculate Percent

The word **percent** comes from the Latin phrase per centum, which means “by the hundred.” So any percent is a number divided by 100. Changing a percent to a fraction is to write the percent in its **fractional form**. To write $n\%$ in its fractional form is to write the percent as the fraction $\frac{n}{100}$.

CHECKPOINT

A percent need not be an integer and does not have to be less than 100.

Example 1 **Rewriting a Percent as a Fraction**

Rewrite the following as fractions:

1. 18%
2. 84%
3. 38.7%
4. 213%

Solution

1. Using the definition and $n = 18$, 18% in fractional form is $\frac{18}{100}$.
2. Using the definition and $n = 84$, 84% in fractional form is $\frac{84}{100}$.
3. Using the definition and $n = 38.7$, 38.7% in fractional form is $\frac{38.7}{100}$.
4. Using the definition and $n = 213$, 213% in fractional form is $\frac{213}{100}$.

Convert Between Percent, Decimal, and Fractional Values

When any calculation with a percent is to be performed, the form of the percent must be changed, either to its fractional form or its **decimal form**. We can change a percent into decimal form by dividing the percent by 100 and representing the result as a decimal.

FORMULA

The decimal form of $n\%$ is found by calculating the decimal value of $n \div 100$.

Example 2 **Converting a Percent to Decimal Form**

Convert the following percents to decimal form:

1. 17%
2. 7%
3. 18.45%

Solution

1. To convert 17% to its decimal form, divide 17 by 100. This moves the decimal two places to the left, resulting in 0.17. The decimal form of 17% is 0.17.

2. To convert 7% to its decimal form, divide 7 by 100. This moves the decimal two places to the left, resulting in 0.07. The decimal form of 7% is 0.07.
3. To convert 18.45% to its decimal form, divide 18.45 by 100. This moves the decimal two places to the left, resulting in 0.1845. The decimal form of 18.45% is 0.1845.

You should notice that, to convert from percent to decimal form, you can simply move the decimal two places to the left without performing the division.

FORMULA

To convert the number x from decimal form to percent, multiply x by 100 and place a percent sign, %, after the number, $(x \times 100)\%$.

Example 3 **Converting the Decimal Form of a Percent to Percent**

Convert each of the following to percent:

1. 0.34
2. 4.15
3. 0.0391

Solution

1. Using the formula and $x = 0.34$, we calculate $(0.34 \times 100)\%$, which gives us 34%.
2. Using the formula and $x = 4.15$, we calculate $(4.15 \times 100)\%$, which gives us 415%.
3. Using the formula and $x = 0.0391$, we calculate $(0.0391 \times 100)\%$, which gives us 3.91%.

You should notice that, to convert from decimal form to percent form, you can simply move the decimal two places to the right without performing the multiplication.

Calculate the Total, Percent, or Part

The word "of" is used to indicate multiplication using fractions, as in "one-fourth of 56." To find "one-fourth of 56" we would multiply 56 by one-fourth. We can think of percents as fractions with a specific denominator—100. So, to calculate "25% of 52," we multiply 52 by 25%. But, first we need to convert the percent to either fractional form (25/100) or decimal form. Using the decimal form of 25% we have 0.25×52, which equals 13.

In this problem, 52 is the **total** or **base**, 25 is the **percentage**, and 13 is **the percentage of** 52, or the **part** of 52. This is sometimes referred to as the **amount**.

FORMULA

The mathematical formula relating the total (base), the percent in decimal form, and the part (amount) is $part = percent \times total$, or, $amount = percent \times base$.

CHECKPOINT

In all calculations, the percent is expressed in decimal form.

Knowing any two of the values in our formula allows us to calculate the third value. In the following example, we know the total and the percent, and are asked to find the percentage of the total.

Example 4 **Finding the Percent of a Total**

1. Determine 70% of 3,500
2. Determine 156% of 720

Solution

1. The total is $x = 3{,}500$, and the percent is $n = 70$. The decimal form of 70% is 0.70. To find the part, or percent of the total, substitute those values into the formula and calculate.

$$\begin{aligned} \text{part} &= \text{percent} \times \text{total} \\ &= 0.70 \times 3500 \\ &= 2450 \end{aligned}$$

   From this, we say that 70% of 3,500 is 2,450.

2. The total is $x = 720$, and the percent is $n = 156$. The decimal form of 156% is 1.56. To find the part, or percent of the total, substitute those values into the formula and calculate.

$$\begin{aligned} \text{part} &= \text{percent} \times \text{total} \\ &= 1.56 \times 720 \\ &= 1{,}123.2 \end{aligned}$$

   From this, we say that 156% of 720 is 1,123.2.

VIDEO

Finding Percent of a Total

In the previous example, we knew the total and the percent and found the part using our formula. We may instead know the percent and the part, but not the total. We can use our formula again to solve for the total.

Example 5 **Finding the Total from the Percent and the Part**

1. What is the total if 35% of the total is 70?
2. What is the total if 10% of the total is 4,000?

Solution

1. **Step 1:** The percent is 35, which in decimal form is 0.35. We were given that 35% of the total is 70, so the part is 70. We are to find the total. Substituting into the formula, we have

$$\begin{aligned} \text{part} &= \text{percent} \times \text{total} \\ 70 &= \quad 0.35 \times \text{total} \end{aligned}$$

Step 2: To find the total, we solve the equation for the total.

$$\begin{aligned} 70 &= 0.35 \times \text{total} \\ \frac{70}{0.35} &= \frac{0.35 \times \text{total}}{0.35} \\ 200 &= \frac{\cancel{0.35} \times \text{total}}{\cancel{0.35}} \\ 200 &= \qquad \text{total} \end{aligned}$$

From this we see that 200 is the total, or, that 35% of 200 is 70.

2. **Step 1:** The percent is 10, which in decimal form is 0.1. We were given that 10% of the total is 4,000, so the part is 4,000. Substituting into the formula, we have

$$\begin{aligned} \text{part} &= \text{percent} \times \text{total} \\ 4,000 &= \quad 0.1 \times \text{total} \end{aligned}$$

Step 2: To find the total, we solve the equation for the total.

$$\begin{aligned} 4,000 &= 0.1 \times \text{total} \\ \frac{4{,}000}{0.1} &= \frac{0.1 \times \text{total}}{0.1} \\ 40,000 &= \frac{\cancel{0.1} \times \text{total}}{\cancel{0.1}} \\ 40,000 &= \qquad \text{total} \end{aligned}$$

From this we see that 40,000 is the total, or that 10% of 40,000 is 4,000.

VIDEO

Finding the Total from the Percent and the Part

Similarly, the percent can be found if the total and the percent of the total (the part) are known. This will result in the decimal form of the percent, so it must be converted to percent form.

Example 6 **Finding the Percent from the Total and the Part**

1. What percent of 500 is 175?
2. What percent of 228 is 155?

Solution

1. **Step 1:** The total is 500, the percent of the total is 175. Substituting into the formula, we have

$$\begin{aligned} \text{part} &= \text{percent} \times \text{total} \\ 175 &= \text{percent} \times 500 \end{aligned}$$

   Step 2: To find the percent, we solve the equation for the percent.

$$\begin{aligned} 175 &= \text{percent} \times 500 \\ \frac{175}{500} &= \frac{\text{percent} \times 500}{500} \\ 0.35 &= \frac{\text{percent} \times \cancel{500}}{\cancel{500}} \\ 0.35 &= \text{percent} \end{aligned}$$

   We see the percent in decimal form is 0.35. Converting from the decimal form yields 35%. We say that 175 is 35% of 500.

2. **Step 1:** The total is 228, the percent of the total is 155. Substituting into the formula, we have

$$\begin{aligned} \text{part} &= \text{percent} \times \text{total} \\ 155 &= \text{percent} \times 228 \end{aligned}$$

   Step 2: To find the percent, we solve the equation for the percent.

$$\begin{aligned} 155 &= \text{percent} \times 228 \\ \frac{155}{228} &= \frac{\text{percent} \times 228}{228} \\ 0.6798 &= \frac{\text{percent} \times \cancel{228}}{\cancel{228}} \\ 0.6798 &= \text{percent} \end{aligned}$$

   We see the percent is 0.6798 (rounded to four decimal places). Converting from the decimal form yields 67.98%. We say that 155 is 67.98% of 228.

VIDEO

Finding the Percent When the Total and the Part Are Known

Solve Application Problems Involving Percents

Percents are frequently used in finance, research, science experiments, and even casual conversation. Understanding these types of values helps when consuming media or discussing finances, for instance. Effectively working with and interpreting numbers and percents will help you become an informed consumer of this information.

In most cases, working through what is presented requires you to identify that you are indeed working with a question of percents, which two of the three values that are related through percents are known, and which of the three values you need to find.

Example 7 **Retention Rate at College**

Justine applies to a medium size university outside her hometown and finds out that the retention rate (percent of students who return for their sophomore year) for the 2021 academic year at the university was 84%. During a visit to the registrar's office, she finds out that 1,350 people had enrolled in academic year 2021. How many students from the academic year 2021 are returning for the 2022 academic year?

Solution

The percent of students who will return for the 2022 academic year (the retention rate) is 84%. The total number of students who enrolled in the 2021 academic year was 1,350. This means the percent is known and the total is known. From this, we can determine the number of students who will return (percent of the total) for the 2022 academic year using the formula part = percent × total. Substituting into the formula and calculating, we find that the number of students that are returning is

$$\begin{aligned} \text{part} &= \text{percent} \times \text{total} \\ &= 0.84 \times 1,350 \\ &= 1,134 \end{aligned}$$

So 1,134 students will return for the 2022 academic year.

Example 8 **Percent of Chemistry Majors**

Cameron enrolls in a calculus class. In this class of 45 students, there are 18 chemistry majors. What percent of the class are chemistry majors?

Solution

In this situation, the percent is to be determined. We know the total number of students, 45, and the part of the students that are chemistry majors, 18. Using that information and the formula part = percent × total, the percent can be found. Substituting and solving, we have

$$\begin{aligned} 18 &= \text{percent} \times 45 \\ \frac{18}{45} &= \frac{\text{percent} \times 45}{45} \\ 0.4 &= \frac{\text{percent} \times \cancel{45}}{\cancel{45}} \\ 0.4 &= \text{percent} \end{aligned}$$

Converting the 0.4 from decimal form, we find that 40% of the students in the calculus class are chemistry majors.

Example 9 **Total Sales and Commission**

Mariel makes a 20% commission on every sale she makes. One week, her commission check is for $153.00. What were her total sales that week?

Solution

In this problem, Mariel's total sales is to be determined. We know the percent she earns is 20%. We also know that her sales commission was $153.00, which is the percent of the total. Using this information and the formula part = percent × total we can find Mariel's total sales. The decimal form of 20% is 0.2. The part, or percent of the total, is 153. Substituting and solving, we obtain

$$\begin{aligned} \text{part} &= \text{percent} \times \text{total} \\ 153 &= 0.2 \times \text{total} \\ \frac{153}{0.2} &= \frac{0.2 \times \text{total}}{0.2} \\ 765 &= \frac{\cancel{0.2} \times \text{total}}{\cancel{0.2}} \\ 765 &= \text{total} \end{aligned}$$

Mariel's total sales were $765.00.

WHO KNEW?

LED Lightbulbs

According to the energy website from the U.S. government, LED lightbulbs use at least 75% less energy than incandescent bulbs. They also last up to 25 times as long as an incandescent bulb. If lighting is a significant percent of your electrical use, replacing all incandescent bulbs with LED bulbs will significantly reduce your electric bill.

For any answer, round to two decimal places, if necessary.

Key Terms

- Percent
- Fractional form

- Decimal form
- Total
- Base
- Percent of the total
- Part
- Amount

Key Concepts

- Know what a percent is as a fraction, a decimal, and as a part of the whole.
- Use the percent equation to find any of the three values that are related by the equation.
- Apply the percent equation in applications.

Video

- Finding Percent of a Total
- Finding the Total from the Percent and the Part
- Finding the Percent When the Total and the Part Are Known

Formulas

part = percent x total

6.2 Discounts, Markups, and Sales Tax

Figure 3: Sale prices are often described as percent discounts.

Learning Objectives

After completing this section, you should be able to:

1. Calculate discounts.

2. Solve application problems involving discounts.
3. Calculate markups.
4. Solve application problems involving markups.
5. Compute sales tax.
6. Solve application problems involving sales tax.

Many people first encounter percentages during a retail transaction such as a percent discount (SALE! 25% off!!), or through sales tax ("Wait, I thought this was \$1.99?"), a report that something has increased by some percentage of the previous value (NOW! 20% more!!). These are examples of percent decreases and percent increases. In this section, we discuss decrease, increase, and then the case of sales tax.

Calculating Discounts

Retailers frequently hold sales to help move merchandise. The sale price is almost always expressed as some amount off the original price. These are **discounts**, a reduction in the price of something. The price after the discount is sometimes referred to as the reduced price or the sale price.

When a reduction is a percent discount, it is an application of percent, which was introduced in Understanding Percent. The formula used was part = percentage × total. In a discount application, the discount plays the role of the part, the percent discount is the percentage, and the original price plays the role of the total.

FORMULA

The formula for a discount based on a percentage is $\text{discount} = \text{percent discount} \times \text{original price}$, with the percent discount expressed as a decimal. The price of the item after the discount is $\text{sale price} = \text{original price} - \text{discount}$.

These are often combined into the following formula

$$\text{sale price} = \text{original price} - \text{percent discount} \times \text{original price} = \text{original price} \times (1 - \text{percent discount})$$

When the original price and the percent discount are known, the discount and the sale price can be directly computed.

Example 1 **Calculating Discount for a Percent Discount**

Calculate the discount for the given price and discount percentage. Then calculate the sale price.

1. Original price = \$75.80; percent discount is 25%
2. Original price = \$168.90; percent discount is 30%

Solution

1. Substituting the values into the formula $\text{discount} = \text{percent discount} \times \text{original price}$, we find that the discount is $\text{discount} = 0.25 \times 75.80 = 18.95$. The

discount is \$18.95. The sale price of the item is then $\text{sale price} = \text{original price} - \text{discount} = 75.80 - 18.95 = 56.85$, or \$56.85.

2. Substituting the values into the formula $\text{discount} = \text{percent discount} \times \text{original price}$, we find that the discount is $\text{discount} = 0.30 \times 168.90 = 50.67$. The discount is \$50.67. The sale price of the item is then $\text{sale price} = \text{original price} - \text{discount} = 168.90 - 50.67 = 118.23$, or \$118.23.

Sometimes the original price and the sale price of an item is known. From this, the percent discount can be computed using the formula $\text{discount} = \text{percent discount} \times \text{original price}$, by solving for the percent discount.

Example 2 **Calculating the Percent Discount from the Original and Sale Prices**

Determine the percent discount based on the given original and sale prices.

1. $\text{Original price} = \$1{,}200.00$; $\text{sale price} = \$900.00$
2. $\text{Original price} = \$36.70$; $\text{sale price} = \$29.52$

Solution

1. **Step 1.** Find the discount. Using the original price and the sale price, we can find the discount with the formula $\text{sale price} = \text{original price} - \text{discount}$. Substituting and calculating, we find the discount to be $900.00 = 1{,}200.00 - \text{discount}$. Solving for the discount gives \$300.00.

Step 2. Find the percent discount. Substituting the discount of \$300.00 and the original price of \$1,200.00, into the formula $\text{discount} = \text{percent discount} \times \text{original price}$, we can find the percent discount.

$$\begin{aligned} 300.00 &= \text{percent discount} \times 1{,}200.00 \\ \frac{300.00}{1{,}200.00} &= \text{percent discount} \\ 0.25 &= \text{percent discount} \end{aligned}$$

Converting to percent form, the percent discount is 25%.

1. **Step 1.** Find the discount. Using the original price and the sale price, we can find the discount with the formula $\text{sale price} = \text{original price} - \text{discount}$. Substituting and calculating, we find the discount to be $29.52 = 36.70 - \text{discount}$. Solving for the discount gives \$7.38.

Step 2. Find the percent discount. Substituting the discount of \$7.38 and the original price of \$36.70, into the formula $\text{discount} = \text{percent discount} \times \text{original price}$, we can find the percent discount.

$$\begin{aligned} 29.52 &= \text{percent discount} \times 36.70 \\ \frac{29.52}{36.70} &= \text{percent discount} \\ 0.2 &= \text{percent discount} \end{aligned}$$

Converting to percent form, the percent discount is 20%.

Sometimes the sale price and the percent discount of an item are known. From this, the original price can be found. To avoid multiple steps, though, the formula that we will use is sale price = original price × (1 − percent discount). The original price can be found by solving this equation for the original price.

Example 3 **Calculating the Original Price from the Percent Discount and Sale Price**

Determine the original price based on the percent discount and sale price.

1. Percent discount 10%; sale price = \$450.00
2. Percent discount 75%, sale price = \$90.00

Solution

1. Using the percent discount and the sale price, we can find the original price with the formula sale price = original price × (1 − percent discount). Substituting and solving for the original price, we find

$$\begin{aligned} \text{sale price} &= \text{original price} \times (1 - \text{percent discount}) \\ 450.00 &= \text{original price} \times (1 - 0.10) \\ 450.00 &= \text{original price} \times (0.90) \\ 500.00 &= \text{original price} \end{aligned}$$

The original price of the item was \$500.00.

1. Using the percent discount and the sale price, we can find the original price with the formulasale price = original price × (1 − percent discount). Substituting and solving for the original price, we find

$$\begin{aligned} \text{sale price} &= \text{original price} \times (1 - \text{percent discount}) \\ 90.00 &= \text{original price} \times (1 - 0.75) \\ 90.00 &= \text{original price} \times (0.25) \\ 360.00 &= \text{original price} \end{aligned}$$

The original price of the item was \$360.00.

Solve Application Problems Involving Discounts

In application problems, identify what is given and what is to be found, using the terms that have been learned, such as discount, original price, percent discount, and sale price. Once you have identified those, use the appropriate formula (or formulas) to find the solution(s).

Example 4 **Determine Discount and New Price a Sale Rack Item**

The sale rack at a clothing store is marked "All Items 30% off." Ian finds a shirt that had an original price of $80.00. What is the discount on the shirt? What is the sale price of the shirt?

Solution

We are asked to find the discount, and the sale price. We know the percent discount is 30%, or 0.30 in decimal form. The original price was $80.

Substituting into the percent discount formula, we find that the discount is discount = percent discount × original price = 0.30 × 80 = 24.

The discount is $24 on that shirt. The sale price is the original price minus the discount, so the sale price is $80 – $24 = $56.

Example 5 **Determine the Percent Discount of a Bus Pass**

An annual pass on the city bus is priced at $240. The student price, though, is $168. What is the percent discount for students for the bus pass?

Solution

We know the original price of the item, $240. We also know the sale price of the item, $168. From this we know the discount is $\$ 240 - \$ 168 = \$ 72$. Substituting these values into the formula discount = percent discount × original price, we can find the percent discount.

$$\begin{aligned} \text{discount} &= \text{percent discount} \times \text{original price} \\ 72 &= \text{percent discount} \times 240 \\ \frac{72}{240} &= \frac{\text{percent discount} \times 240}{240} \\ 0.3 &= \text{percent discount} \end{aligned}$$

The student percent discount on the bus pass is 30%.

Example 6 **Finding the Original Price of a New Pair of Tires**

Kendra's car developed a flat, and the tire store told her that two tires had to be replaced. She got a 10% discount on the pair of tires, and the sale price came to $189.00. What was the original price of the tires?

Solution

Using the percent discount and the sale price, we can find the original price with the formulasale price = original price × (1 − percent discount). Substituting and solving for the original price, we find

$$\begin{aligned} \text{sales price} &= \text{original price} \times (1 - \text{percent discount}) \\ 189.00 &= \text{original price} \times (1 - 0.10) \\ 189.00 &= \text{original price} \times (0.90) \\ 210.00 &= \text{original price} \end{aligned}$$

The original price of the two tires Kendra bought was \$210.00.

VIDEO

Computing Price Based on a Percent Off Coupon

WORK IT OUT

There are cases where retailers allow multiple discounts to be applied. However, it is rare that the discount percentages are added together. For example, if you have a 15% coupon and qualify for a 20% price reduction, the retailer typically does not add those two percentages together to determine the new price. The retailer instead applies one discount, then applies the second discount to the price obtained after the first discount was deducted.

Research the original prices of two different laptops offered by one retail outlet. Assume you will receive a student discount of 12% and your outlet of choice is having a 15% off sale on all laptops.

For each laptop:

1. List the original price and calculate the price after applying the student discount (12%) only.
2. Then find the price after applying the sale discount (15% off) to the price found in Step 1.
3. Determine the total saved on the laptop and what percent discount the total savings represents.
4. Now, apply the discounts in reverse order (first the sale discount, then the student discount).
5. Note anything interesting about your findings.

Calculate Markups

When retailers purchase goods to sell, they pay a certain price, called the **cost**. The retailer then charges more than that amount for the goods. This increase is called the **markup**. This selling price, or **retail price**, is what the retailer charges the consumer in order to pay their own costs and make a profit. Markup, then is very similar to discount, except we add the markup, while we subtract the discount.

FORMULA

The formula for a markup based on a percentage is markup = percent markup × cost, with the percent markup expressed as a decimal. The price of the item after the markup is retail price = cost + markup.

These are often combined into the following formula

$$\text{retail price} = \text{cost} + \text{percent markup} \times \text{cost} = \text{cost} \times (1 + \text{percent markup})$$

It should be noted that the formulas used for a markup are very similar to those for a discount, with addition replacing the subtraction.

Example 7 **Determining the Retail Price Based on the Cost and the Percent Markup**

Calculate the markup for the given cost and markup percentage. Then calculate the retail price.

1. Cost = $62.00; percent markup is 15%
2. Cost = $750.00; percent markup is 45%

Solution

1. Substituting the values into the formula markup = percent markup × cost, we find that the markup is markup = 0.15 × 62.00 = 9.30. The markup is $9.30.
   The retail price of the item is then retail price = cost + markup, or $62.00 + $9.30 = $71.30.
2. Substituting the values into the formula markup = percent markup × cost, we find that the markup is markup = 0.45 × 750.00 = 337.50. The markup is $337.50.
   The retail price of the item is then retail price = cost + markup, or $750.00 + $337.50 = $1,087.50.

Sometimes the cost and the retail price of an item are known. From this, the percent markup can be computed using the formula markup = percent markup × cost, by solving for the percent markup.

Example 8 **Calculating the Percent Markup from the Cost and Retail Price**

Determine the percent markup based on the given cost and retail price. Round percentages to two decimal places.

1. Cost = $90.00; retail price = $103.50
2. Cost = $5.20; retail price = $9.90

Solution

1. **Step 1:** Using the cost and the retail price, we can find the markup with the formula retail price = cost + markup. Substituting and calculating, we find the markup to be 103.50 = 90.00 + markup. Solving for the markup gives $13.50.

Step 2: After substituting the markup, \$13.50, and the original price, \$90.00, into the formula markup = percent markup × cost, we can find the percent markup.

$$\begin{aligned} 13.50 &= \text{percent markup} \times 90.00 \\ \frac{13.50}{90.00} &= \text{percent markup} \\ 0.15 &= \text{percent markup} \end{aligned}$$

Converting to percent form, the percent markup is 15%.

1. **Step 1:** Using the cost and the retail price, we can find the markup with the formula retail price = cost + markup. Substituting and calculating, we find the markup to be 9.90 = 5.20 + markup. Solving for the markup gives \$4.70.

Step 2: After substituting the markup, \$4.70, and the original price, \$5.20, into the formula markup = percent markup × cost, we can find the percent markup.

$$\begin{aligned} 4.70 &= \text{percent markup} \times 5.20 \\ \frac{4.70}{5.20} &= \text{percent markup} \\ 0.9038 &= \text{percent markup} \end{aligned}$$

Converting to percent form, the percent markup is 90.38%.

Sometimes the retail price and the percent markup of an item are known. From this, the cost can be found. To avoid multiple steps, though, the formula that we will use is retail price = cost × (1 + percent markup). The cost can be found by solving this equation for the cost.

Example 9 **Calculating the Cost from the Percent Markup and Retail Price**

Determine the cost based on the percent markup and retail price.

1. Percent markup 20%; retail price = \$10.62
2. Percent markup 125%; retail price = \$26.55

Solution

1. Using the percent markup and the retail price, we can find the cost with the formula retail price = cost × (1 + percent markup). Substituting and solving for the cost, we find

$$\begin{aligned} \text{retail price} &= \text{cost} \times (1 + \text{percent markup}) \\ 10.62 &= \text{cost} \times (1 + 0.2) \\ 10.62 &= \text{cost} \times (1\ .2) \\ 8.85 &= \text{cost} \end{aligned}$$

The cost of the item was \$8.85.

1. Using the percent markup and the retail price, we can find the cost with the formularetail price = cost × (1 + percent markup). Substituting and solving for the original price, we find

$$\begin{aligned} \text{retail price} &= \text{cost} \times (1 + \text{percent markup}) \\ 26.55 &= \text{cost} \times (1{+}2\ .25) \\ 26.55 &= \text{cost} \times (3\ .25) \\ 11.80 &= \text{cost} \end{aligned}$$

The cost of the item was \$11.80.

Solve Application Problems Involving Markups

As before when working with application problems, be sure to look for what is given and identify what you are to find. Once you have evaluated the problem, use the appropriate formula to find the solution(s). These application problems address markups.

Example 10 **Determine Retail Price of a Power Bar**

Janice works at a convenience store near campus. It sells protein bars at a 60% markup. If a bar costs the store \$1.30, how much is the retail price at the convenience store?

Solution

We are asked to find the retail price. We know the percent markup is 60%. The cost of the bar was \$1.30. Substituting into the percent markup formula, we find that the markup is markup = percent markup × cost = 0.60 × 1.30 = 0.78 . The markup is \$0.78 on that protein bar. The retail price is the cost plus the markup, so the retail price is retail price = cost + markup = 1.30 + 0.78 = 2.08. The retail price is \$2.08.

Example 11 **Determine the Percent Markup of a Phone**

Javi began working at a phone outlet. In a recent shipment, he noticed that the cost of the phone to the store was \$480.00. The phone sells for \$840.00 in the store. What is the percent markup on the phone?

Solution

We know the cost of the phone, \$480. We also know the retail price of the phone, \$840.00. From this we know the markup is \$ 840.00 − \$ 480.00 = \$ 360.00. Substituting these values into the formula markup = percent markup × cost, we can find the percent markup.

$$\begin{aligned} \text{markup} &= \text{percent markup} \times \text{cost} \\ 360 &= \text{percent markup} \times 480 \\ \frac{360}{480} &= \frac{\text{percent markup} \times 480}{480} \\ 0.75 &= \text{percent markup} \end{aligned}$$

The markup on the phone is 75%.

Example 12 **Finding the Cost of a T-Shirt**

Bob decided to order a t-shirt for his gaming friend online for \$29.50. He knows the markup on such t-shirts is 18%. What was the t-shirt's cost before the markup?

Solution

Using the percent markup and the retail price, \$29.50, we can find the cost with the formula retail price = original price × (1 + percent markup). Substituting and solving for cost we find

$$\begin{aligned} \text{retail price} &= \text{cost} \times (1 + \text{percent markup}) \\ 29.50 &= \text{cost} \times (1{+}0\,.18) \\ 29.50 &= \text{cost} \times (1\,.18) \\ 25.00 &= \text{cost} \end{aligned}$$

The cost of the t-shirt was \$25.00.

Compute Sales Tax

Sales tax is applied to the sale or lease of some goods and services in the United States but is not determined by the federal government. It is most often set, collected, and spent by individual states, counties, parishes, and municipalities. None of these sales tax revenues go to the federal government.

For example, North Carolina has a state sales tax of 4.75% while New Mexico has a state sales tax of 5%. Additionally, many counties in North Carolina charge an additional 2% sales tax, bringing the total sales tax for most (72 of the 100) counties in North Carolinians to 6.75%. However, in Durham, the county sales tax is 2.25% plus an additional 0.5% tax used to fund public transportation, bringing Durham County's sales tax to 7%. To find the sales tax in a particular place, then, add other locality sales taxes to the base state sales tax rate.

How much we pay in sales tax depends on where we are, and what we are buying.

To determine the amount of sales tax on taxable purchase, we need to find the product of the purchase price, or marked price, and the sales tax rate for that locality.

FORMULA

To calculate the amount of sales tax paid on the purchase price in a locality with sales tax given in decimal form, calculate sales tax = purchase price × tax rate The total price is then Total price = purchase price + purchase price × tax rate = purchase price × (1 + tax rate)

CHECKPOINT

When the sales tax calculation results in a fraction of a penny, then normal rounding rules apply, round up for half a penny or more, but round down for less than half a penny.

You should notice that this the same as markup, except using a different term. Sales tax plays the role of markup, the purchase price plays the role of cost, and the tax rate plays the role of percent markup. This means all the strategies developed for markups apply to this situation, with the changes indicated.

Example 13 **Sales Tax in Kankakee Illinois**

The sales tax in Kankakee, Illinois, is 8.25%. Find the sales tax and total price of items based on the purchase price listed.

1. Purchase price = $428.99
2. Purchase price = $34.88

Solution

1. The sales tax is found using sales tax = purchase price × tax rate. The purchase price is $428.99 and the tax rate is 8.25%. Substituting and calculating, the sales tax is sales tax $= \$\,428.99 \times 0.0825 = \$\,35.391675$. The sales tax needs to be rounded off. Since the third decimal place (fraction of a penny) is 1, we round down and the sales tax is $35.39. The total price is the sales tax plus the purchase price, so is $\$\,428.99 + \$\,35.88 = \$\,464.87$.
2. The sales tax on the item is found using sales tax = purchase price × tax rate. The purchase price is $34.88 and the tax rate is 8.25%. Substituting and calculating, the sales tax is sales tax $= \$\,34.88 \times 0.0825 = \$\,2.8776$. The sales tax needs to be rounded off. Since the third decimal place (fraction of a penny) is 7, we round up and the sales tax is $2.88. The total price of the item is the sales tax plus the purchase price, so is $\$\,34.88 + \$\,2.88 = \$\,37.76$.

As before, the information available might be different than only the purchase price and the sales tax rate. In these cases, use either sales tax = purchase price × tax rate or Total price = purchase price × (1 + tax rate) and solve for the indicated tax, price, or rate. These problems mirror those for percent markup.

Be aware, almost all sales tax rates are structured as full percentages, or half percent, or one-quarter percent, or three-quarter percent. This means the decimal value of the sales tax rate, written as a percent, will be either 0, as in 5.0%, 5 as in 7.5%, 25 as in 3.25%, or 75 as in 4.75%. When rounding for the sales tax percentage, be sure to use this guideline.

Example 14 **Calculating the Sales Tax from the Purchase Price and the Total Price**

Find the sales tax rate for the indicated purchase price and total price. Round using the guideline for sales tax percentages.

1. Purchase price = $329.50; total price = $354.21
2. Purchase Price = $13.77; total price = $14.39

Solution

1. **Step 1.** Find the sales tax paid. First, the amount of sales tax must be found. Subtracting the purchase price from the total price, the amount of sales tax is $24.71.
   Step 2. Find the sales tax rate. Using the purchase price, the sales tax, and the formula sales tax = purchase price × tax rate, the sales tax rate can be found. Substituting and solving yields

$$\begin{aligned} \text{Sales Tax} &= \text{purchase price} \times \text{tax rate} \\ \$\,24.71 &= \$\,329.50 \times \text{tax rate} \\ \frac{\$\,24.71}{\$\,329.50} &= \text{tax rate} \\ 0.07499 &= \text{tax rate} \end{aligned}$$

Keeping in mind the guideline for rounding sales tax rate, the sales tax rate is 7.5%.

1. **Step 1.** Find the sales tax paid. First, the amount of sales tax must be found. Subtracting the purchase price from the total price, the amount of sales tax is $0.62.

Step 2. Find the sales tax rate. Using the purchase price, the sales tax, and the formula sales tax = purchase price × tax rate, the sales tax rate can be found. Substituting and solving yields

$$\begin{aligned} \text{sales tax} &= \text{purchase price} \times \text{tax rate} \\ \$\,0.62 &= \$\,13.77 \times \text{tax rate} \\ \frac{\$\,0.62}{\$\,13.77} &= \text{tax rate} \\ 0.04503 &= \text{tax rate} \end{aligned}$$

Keeping in mind the guideline for rounding sales tax rate, the sales tax rate is 4.5%.

Example 15 **Calculating the Purchase Price from the Sales Tax and Total Price**

Find the purchase price for the indicated sales tax rate and total price.

1. Sales tax rate = 5.75%; total price = $36.56
2. Sales tax rate = 4.25%; total price = $97.17

Solution

1. When the sales tax rate and the total price are known, the formula total price = purchase price × (1 + tax rate) can be used to find the purchase price. Substituting the tax rate and total price into the formula and solving, we find

$$\begin{aligned} \text{Total price} &= \text{purchase price} \times (1 + \text{tax rate}) \\ \$\,36.56 &= \text{purchase price} \times (1 + 0.0575) \\ \frac{\$\,36.56}{1.0575} &= \text{purchase price} \\ \$\,34.57 &= \text{purchase price} \end{aligned}$$

The purchase price, the price before tax, was \$34.57.

1. When the sales tax rate and the total price are known, the formula total price = purchase price × (1 + tax rate) can be used to find the purchase price. Substituting the tax rate and total price into the formula and solving, we find

$$\begin{aligned} \text{Total price} &= \text{purchase price} \times (1 + \text{tax rate}) \\ \$\,97.17 &= \text{purchase price} \times (1 + 0.0425) \\ \frac{\$\,97.17}{1.0425} &= \text{purchase price} \\ \$\,93.21 &= \text{purchase price} \end{aligned}$$

The purchase price, the price before tax, was \$93.21.

Solve Application Problems Involving Sales Tax

Solving problems involving sales tax follows the same ideas and steps as solving problems for markups. But here we will use the following formula:

$$\text{total price} = \text{purchase price} + \text{ sales tax}$$

We can also use the formula:

$$\text{total price} = \text{purchase price} \times (1 + \text{sales tax rate})$$

This can be seen in the following examples.

Example 16 **Compute Sales Tax for Denver, Colorado**

The sales tax rate in Denver Colorado is 8.81%. Keven buys a TV in Denver, and the purchase price (before taxes) is \$499.00. How much will Keven pay in sales tax and what will be the total amount he spends when he buys the TV?

Solution

The sales tax rate in Denver is 8.81%. To find the sales tax Keven will pay, find 8.81% of the purchase price. In decimal form, that sales tax rate is 0.0881. Using the formula and substituting 499.00 for purchase price, we find that Keven will pay purchase price $\times$ tax rate = $\$499 \times 0.0881 = \$43.96$ in sales tax for the TV.

The total price that Keven will pay is the purchase price plus the sales tax, or $\$499.00 + \$43.96 = \$542.96$.

Example 17 **Compute Sales Tax for Austin, Texas**

Jillian visits Austin, Texas, and purchases a new set of weights for her home. She spends, including sales tax, $467.64. The sales tax rate in Austin Texas is 8.25%. How much of the total price is sales tax?

Solution

The sales tax paid for this purchase is the difference in the total price and the purchase price. We know the total price is $467.64. We also know the sales tax rate, which is 8.25%. In decimal form, this is 0.0825. Using these values and the formula total price = purchase price $\times$ (1 + tax rate) to find the purchase price.

$$\begin{aligned} \text{total price} &= \text{purchase price} \times (1 + \text{tax rate}) \\ \$467.64 &= \text{purchase price} \times (1 + 0.0825) \\ \$467.64 &= \text{purchase price} \times (1.0825) \\ \$432 &= \text{purchase price} \end{aligned}$$

Knowing both the total price and the now the purchase price, we can find the difference, which is the sales tax.

The total price was $467.64. The purchase price was $432. The difference of the total price and the purchase price, or the sales tax, is then $467.64 − $432.00, which is $35.64. Jillian pays $35.64 in sales tax.

VIDEO

Finding Sales Tax Percentage

WHO KNEW?

West Virginia was the first state to impose a sales tax. This happened on May 3, 1921.

NOTE

Look up your locality on this website that lists standard state-level sales tax rates and compare the sales tax structure in your state to two nearby states (for the lower 48) and for any two states (Alaska and Hawaii).

Key Terms

- Discount
- Cost

- Markup
- Retail price

Key Concepts

- Discounts are markdowns from an original price.
- Mark-ups are increases to the price paid by a retailer to cover their costs.
- be able to calculate the markup based on a percentage of the cost
- Sales taxes vary from state to state and often county to county.
- Retail prices, sales prices and percent discounts can be calculated if the other two values are known.
- Original costs, retail prices, and percent markup can be calculated if the other two values are known.
- In calculations, sales tax acts like a markup.

Videos

- Computing Price Based on a Percent Off Coupon
- Finding Sales Tax Percentage

Formulas

$$\text{discount} = \text{percent discount} \times \text{original price}$$

$$\text{sale price} = \text{original price} - \text{discount}$$

$$\text{sale price} = \text{original price} - \text{percent discount} \times \text{original price} = \text{original price} \times (1 - \text{percent discount})$$

$$\text{markup} = \text{percent markup} \times \text{cost}$$

$$\text{retail price} = \text{cost} + \text{markup}$$

$$\text{retail price} = \text{cost} + \text{percent markup} \times \text{cost} = \text{cost} \times (1 + \text{percent markup})$$

$$\text{sales tax} = \text{purchase price} \times \text{tax rate}$$

$$\text{Total price} = \text{purchase price} + \text{purchase price} \times \text{tax rate} = \text{purchase price} \times (1 + \text{tax rate})$$

6.3 Simple Interest

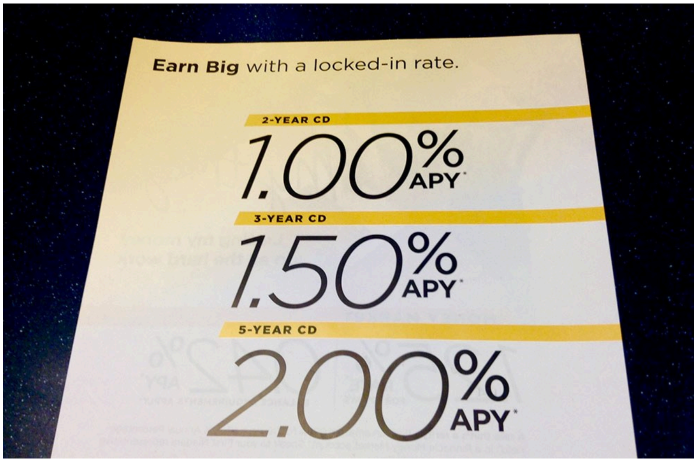


Figure 4: Interest is how savings earns money.

Learning Objectives

After completing this section, you should be able to:

1. Compute simple interest.
2. Understand and compute future value.
3. Compute simple interest loans with partial payments.
4. Understand and compute present value.

There is truth in the phrase “You need to have money to make money.” In essence, if you have money to lend, you can lend it at a cost to a borrower and make money on that transaction.

When money is borrowed, the person borrowing the money (borrower) typically has to pay the person or entity that lent the money (the lender) more than the amount of money that was borrowed. This extra money is the **interest** that is to be paid. Interest is sometimes referred to as the cost to borrow, the cost of the loan, or the finance cost.

This idea also applies when someone deposits money in a bank account or some other form of investment. That person is essentially lending the money to the bank or company. The money earned by the depositor is also called interest. The interest is typically based on the amount borrowed, or the **principal**.

The pairing of borrower and lender can take various forms. The borrower may be a consumer using a credit card or taking out a loan from a bank, the lender. Companies also borrow from lending banks. Someone who invests in a company’s stock is the lender in this case; the company is essentially the borrower.

In this section, we examine the basic building block of interest paid on loans and borrowed credit and also the returns on investments like bank accounts, simple interest.

Compute Simple Interest

Let's get some terminology understood. Interest to be paid by a borrower is often expressed as an **annual percentage rate**, which is the percent of the principal that is paid as interest for each year the money is borrowed. This means that the more that is borrowed, the more that must be paid back. Sometimes, the interest to be paid back is **simple interest**, which means that the interest is calculated on the amount borrowed only.

The length of time until the loan must be paid off is the **term** of the loan. The date when the loan must be paid off is when the loan is **due**. The day that the loan is issued is the **origination date**. We'll put this terminology to use in the following examples. Note that in this section we will use letters, called variables, to represent the different parts of the formulas we'll be using. This will help keep our formulas and calculations manageable.

Simple Interest Loans with Integer Year Terms

Calculating simple interest is similar to the percent calculations we made in Understanding Percent and Discounts, Markups, and Sales Tax, but must be multiplied by the term of the loan (in years, if dealing with an annual percentage rate).

FORMULA

The simple interest, I, to be paid on a loan with annual interest rate r for a number of years (term of the loan) t, with principal P, is found using $I = P \times r \times t$, where the decimal form of the interest rate, r, is used. The total repaid, then is $T = P + I$ or, more directly, $T = P + P \times r \times t$. This total is often referred to as the loan **payoff amount**, or more simply just the payoff.

When the annual interest rate, the principal, and the number of years that the money is borrowed is known, the interest to be paid can be found and from there the total to be repaid can be calculated.

CHECKPOINT

Be aware, interest paid to a lender is almost uniformly rounded up to the next cent.

Example 1 **Simple Interest on Loans with Integer Year Terms**

Calculate the simple interest to be paid on a loan with the given principal, annual percentage rate, and number of years. Then, calculate the loan payoff amount.

1. Principal $P = \$4{,}000$, annual interest rate $r = 5.5\%$, and number of years $t = 4$
2. Principal $P = \$14{,}800$, annual interest rate $r = 7.9\%$, and number of years $t = 7$

Solution

1. Substitute the principal $P = \$4{,}000$, the decimal form of the annual interest rate $r = 0.055$, and number of years $t = 4$ into the formula for simple interest, and calculate.
   $I = P \times r \times t = 4,000 \times 0.055 \times 4 = 880$.
   The simple interest, or cost of the loan, to be paid on the loan is \$880.

The loan payoff amount, or the total to be repaid, is $T = P + I = 4,000 + 880 = 4,880$, or $4,880.00.

2. Substitute the principal $P = \$14{,}800$, the decimal form of the annual interest rate $r = 0.079$, and number of years $t = 7$ into the formula for simple interest, and calculate.
$I = P \times r \times t = 14,800 \times 0.079 \times 7 = 8,184.4$.
The simple interest, or cost to borrow, to be paid on the loan is $8,184.40.
The loan payoff amount, or the total to be repaid, is $T = P + I = 14,800 + 8,184.4 = 22,984.4$, or $22,984.40.

Example 2 **Simple Interest Equipment Loan**

Riley runs an auto repair shop, and needs to purchase a new brake lathe, which costs $11,995. She takes out a two-year, simple interest loan at an annual interest rate of 14.9%. How much interest will she pay and how much total will she repay on the loan?

Solution

Step 1. Determine the variables, or parts of the formula. The principal P is the cost of the brake lathe, so $P = \$11{,}995$. The interest rate Riley pays is 14.9%, or $r = 0.149$ in decimal form. The length of the loan is two years, so $t = 2$. We are first asked to find I, the interest Riley will pay.

Step 2. Substitute the known variables into the formula for simple interest $I = P \times r \times t$ and solve for I.

From Step 1 we have $I = P \times r \times t = 11995 \times 0.149 \times 2 = 3,574.51$.

This tells us that the simple interest, or cost to borrow, to be paid on the loan is $3,574.51.

Step 3. Use the formula $T = P + I$ to determine the total amount Riley will repay, T.

The total to be repaid is $T = P + I = 11,995 + 3,574.51 = 15,569.51$, or $15,569.51.

Simple Interest Loans with Other Lengths of Terms

In the previous example and Your Turn exercise, the loans were paid back in one payment after an integer number of years. However, there are also loans lasting a length of time not equal to an integer number of years (like 1, 2, or 3 years or more), but in a number of months (like 4 months, 18 months, and so on). What model would apply to these situations?

When the loan is paid back after a term that is not an integer number of years but is instead a number of months, the term of the loan, or time, t, is expressed as a fraction of the year. So for a 2-month loan, the time, in years, is $2/12 = 1/6$. For a 5-month loan, the time in years is $5/12$. For an 18-month term, the term in years is $18/12 = 1.5$.

Example 3 **Loan to Purchase Equipment**

Abeje needs a loan to purchase equipment for the gym she is going to open. She visits the bank and secures a 4-month loan of $20,000. Her annual percentage rate is 6.75%. How much interest will Abeje pay and what is her loan payoff amount?

Solution

Abeje's loan is for $20,000, so her principal is $P = 20{,}000$. The interest rate Abeje will pay is 6.75%, or $r = 0.0675$ in decimal form. The length of the loan is 4 months, so $t = \frac{4}{12}$. Substituting these in the formula for simple interest, we find her interest to be $I = P \times r \times t = 20,000 \times 0.0675 \times \frac{4}{12} = 450$

The simple interest, or cost to borrow, to be paid on the loan is $450.00.

The payoff is $T = P + I = 20,000 + 450 = 20,450$, or $20,450.00.

Those examples dealt in months. However, some loans are for days only (45 days, 60 days, 120 days). In such cases, we find the daily interest rate. The fraction we will use for the daily interest rate is the interest rate (as a decimal) divided by 365. This may be referred to as Actual/365. In order to find the term of the loan, divide the number of days in the term of the loan by 365.

FORMULA

To determine the interest, I, on a loan with term t expressed in days, with principal of P, and interest rate in decimal form of r, calculate $I = P \times \frac{r}{365} \times t$. Here, $\frac{r}{365}$ represents the daily interest rate.

Alternately, the above formula is equivalent to $I = P \times r \times \frac{t}{365}$, where the interest rate remains an annual rate, but the time is expressed as a fraction of the year.

WHO KNEW?

It seems reasonable to use 365 as the number of days in the year, since there are 365 days in most years. However, sometimes, banks have used (and continue to use) 360 as the number of days in a year. They may also treat all months as if they have 30 days. These differences lead to (sometimes small) differences in how much interest is paid. Since the number of days is in the denominator, a smaller denominator (360) will result in larger numbers (interest) that is 365 is used for the denominator. See this page from ACRE for a comparison.

Example 4 Loan for Moving Costs

David plans to move his family from Raleigh, North Carolina to Tempe, Arizona. His company will reimburse (pay after the move) David for the move. David does research and determines that movers will cost $5,600 to move his family's belongings to Tempe. He takes out a simple interest, 45-day loan at 11.75% interest to pay this cost. How much interest will be paid on this 45-day loan, and what is David's loan payoff amount?

Solution

This loan is in terms of days, so we will use the formula $I = P \times \frac{r}{365} \times t$, where t is the number of days and r is the annual interest rate.

The principal for the loan is the moving cost, or $P = 5{,}600$. The annual interest rate that David will pay is 11.75%, which in decimal is 0.1175. The length of time for the loan is 45 days, so $t = 45$.

Substituting these values into the formula and calculating, we find that the interest to be paid is $I = 5,600 \times \frac{0.1175}{365} \times 45 = 81.13$, or $81.13 (remember, interest is almost always rounded up to the next cent).

The payoff for the loan is $5,681.13.

Understand and Compute Future Value

Money can be invested for a specific amount of time and earn simple interest while invested. The terminology and calculations are the same as we've already seen. However, instead of the total to be paid back, the investor is interested in the total value of the investment after the interest is added. This is called the **future value** of the investment.

FORMULA

The future value, FV, of an investment that yields simple interest is $FV = P + I = P + P \times r \times t$, where P is the principal (amount invested at the start), r is the annual interest rate in decimal form, and t is the length of time the money is invested. The time t will be an integer if the term of the deposit is an integer number of years, will be number of months/12 if the term is in months, will be actual/365 if the deposit is for a number of days.

Example 5 **Simple Interest on a Deposit**

In the following, determine how much interest was earned on the investment and the future value of the investment, if the investment yields simple interest.

1. Principal is $1,000, annual interest rate is 2.01%, and time is 5 years
2. Principal is $5,000, annual interest rate is 1.85%, and time is 30 years
3. Principal is $10,000, annual interest rate is 1.25%, and time is 18 months
4. Principal is $7,000, annual interest rate is 3.26%, and time is 100 days

Solution

1. The principal is $P = \$1{,}000$, the annual interest rate, in decimal form, is 0.0201, and the term is 5 years, or $t = 5$. Since the term is an integer number of years, the interest earned on the investment is $I = P \times r \times t = 1,000 \times 0.0201 \times 5 = 100.5$, or the interest earned was $100.50.
   To find the future value, we use the formula $FV = P + I$. Substituting the values and calculating, we find the future value of the investment to be $FV = P + I = 1,000 + 100.5 = 1100.5$. The future value of the investment at the end of 5 years is $1,100.50. Notice that the future value could have been calculated directly with $FV = P + P \times r \times t$

2. The principal is $P = \$5{,}000$, the annual interest rate, in decimal form, is 0.0185, and the term is 30 years, or $t = 30$. Since the term is an integer number of years, the interest earned on the investment is $I = P \times r \times t = 5,000 \times 0.0185 \times 30 = 2,775$, or the interest earned was $2,775.00. To find the future value, we use the formula $FV = P + I$. Substituting the values and calculating, we find the future value of the investment to be $FV = P + I = 5,000 + 2,775 = 7,775$. The future value of the investment at the end of 30 years is $7,775.00.

3. The principal is $P = \$10{,}000$, the annual interest rate, in decimal form, is 0.0125, and the term is 18 months. Since the term is in months, we have to write the months in terms of years. For 18 months, we use 18/12 as t. The interest earned on the investment is $I = P \times r \times t = 10,000 \times 0.0125 \times \frac{18}{12} = 187.5$, or the interest earned was $187.50. To find the future value, we use the formula $FV = P + I$. Substituting the values and calculating, we find the future value of the investment to be $FV = P + I = 10,000 + 187.5 = 10,187.5$. The future value of the investment at the end of 18 months is $10,187.50.

4. Principal is $7,000, annual interest rate is 3.26%, and time is 100 days. The principal is $P = \$7{,}000$, the annual interest rate, in decimal form, is 0.0326, and the term is 100 days. Since the term is in days, we have to write the time using actual/365, or $t = 100/365$. The interest earned on the investment is $I = P \times r \times t = 7,000 \times 0.0326 \times \frac{100}{365} = 62.52$, or the interest earned was $62.52. To find the future value, we use the formula $FV = P + I$. Substituting the values and calculating, we find the future value of the investment to be $FV = P + I = 7,000 + 62.52 = 7,062.52$. The future value of the investment at the end of 100 days is $7,062.52.

You may have noticed that for these problems, the future value was rounded down. When the future value is paid, the amount is typically rounded down.

A certificate of deposit (CD) is a savings account that holds a single deposit (the principal) for a fixed term at a fixed interest rate. Once the term of the CD is over, the CD may be redeemed (cashed in or withdrawn) and the owner of the CD receives the original principal plus the interest earned. The deposit often cannot be withdrawn until the term is up; if it can be withdrawn early, there is often a penalty imposed to do so.

Example 6 **Certificate of Deposit**

Jonas deposits $2,500 in a CD bearing 3.25% simple interest for a term of 3 years. When he redeems his CD at the end of the 3 years, how much will he receive?

Solution

This is a future value example. We know that $P = \$2{,}500$ is the amount deposited. The annual simple interest rate in decimal form is $r = 0.0325$. The term of the investment is $t = 3$ years.

Substituting those values into the future value formula, we have $FV = P + P \times r \times t = 2,500 + 2,500 \times 0.0325 \times 3 = 2,500 + 243.75 = 2,743.75$.

When the CD is redeemed, Jonas will receive $2,743.75.

WORK IT OUT

The reason CD (certificate of deposit) rates look so small is because they are extremely safe investments. Though overall interest rates for CDs change over time and individual returns vary with the terms of the CD, investors are offered predictable interest income for their investments.

To investigate this yourself, search online to determine the strengths and weaknesses of CDs (investopedia.com offers good, basic information on investing). Then, online, identify five national banks and two local banks who offer CDs.

- Track the interest rates for the CDs at various terms (1 year, 3 years, 5 years) for each of the banks you found that offer CDs.
- Calculate the amount of interest earned for a $10,000 deposit for each CD at each of the terms.
- Compare the results from the various banks, CDs, and terms and decide which is the best investment. You may want to consider both the length of time that the money is locked up, and the return.

Paying Simple Interest Loans with Partial Payments

In every example above, there was one payment for the loan, or one withdrawal for the investment. However, for many loans (house, car, in-ground swimming pool), the loan will be paid back in two or more payments. Such a payment is called a **partial payment**, because they only pay off part of the loan.

When a partial payment is made, some of the payment pays for the principal, but the rest of the payment pays for interest on the principal. When making the first partial payment, the interest is calculated on the principal for the time between the origination date of the loan and the date of the payment. If another partial payment is made, the interest is calculated based on the remaining principal and the time between the previous partial payment and the current partial payment date.

Example 7 **Interest Paid in a Partial Payment on a Loan**

1. A simple interest loan for $6,500 is taken out at 12.6% annual percentage rate. A partial payment is made 45 days into the loan period. How much of the partial payment will be for interest?

2. A simple interest loan for \$13,700 is taken out at 6.55% annual interest rate. A partial payment is to be made after 60 days. How much of the partial payment will be for interest?

Solution

1. To find the interest paid in this partial payment, we calculate the interest on the principal for the time between the origination of the loan and the payment day, or 45 days.
   The principal is \$6,500. The annual interest rate, in decimal form, is 0.126.
   The interest paid for 45 days is found by substituting the values for principal P, rate r, and time t into the formula $I = P \times \frac{r}{365} \times t$.
   Calculating, we have $I = P \times \frac{r}{365} \times t = 6,500 \times \frac{0.126}{365} \times 45 = 100.9726$. Rounding up, the portion of the partial payment that will be paid for interest is \$100.98.

2. To find the interest paid in this partial payment, we calculate the interest on the principal for the time between the origination of the loan and the payment day, or 60 days.
   The principal is \$13,700. The annual interest rate, in decimal form, is 0.0655.
   The interest paid for those 60 days is found by substituting those values into the formula $I = P \times \frac{r}{365} \times t$. Calculating, we have $I = P \times \frac{r}{365} \times t = 13,700 \times \frac{0.0655}{365} \times 60 = 147.5096$. Rounding up, the portion of the partial payment that will be paid for interest is \$147.51.

Remaining Balance

The previous examples demonstrated how to determine the interest paid in a partial payment. Using this, we can determine the remaining balance after a partial payment.

Step 1: determine the amount of the payment, P, that is applied to interest, I.

Step 2: subtract the amount paid in interest from the payment, $(P - I)$. This is the amount applied to the balance.

Step 3: subtract the amount applied to the balance (the value obtained in Step 2) from the balance of the loan, $B - (P - I)$. This is the remaining balance after the partial payment.

Example 8 **Determining the Remaining Balance on a Loan After a Partial Payment**

1. A simple interest loan for \$45,500 is taken out at 11.8% annual percentage rate. A partial payment of \$20,000 is made 50 days into the loan period. After this payment, what will the remaining balance of the loan be?

2. A simple interest loan for \$150,000 is taken out at 5.85% annual percentage rate. A partial payment of \$50,000 is made 70 days into the loan period. After this payment, what will the remaining balance of the loan be?

Solution

1. The principal is \$45,500, which will be treated as the balance, B, of the loan. The annual simple interest rate, in decimal form, is 0.118. The time is $t = 50$ days.
   Step 1: Determine the amount of the partial payment that is applied to interest. To find this, substitute the values above into the formula $I = P \times \frac{r}{365} \times t$ and calculate. Calculating, the amount of the payment that is applied to interest is $I = 45,500 \times \frac{0.118}{365} \times 50 = 735.4795$. Rounding up, we have $I = \$735.48$.
   Step 2: The amount of the payment that is to be applied to the balance of the loan is partial payment minus the amount of the partial payment that is applied to the interest. The payment is \$2,000. The amount that is applied to the balance is $P - I = \$\,20,000 - \$\,735.48 = \$\,19,264.52$.
   Step 3: The remaining balance is found by subtracting the amount applied to the balance from the previous balance, or $B - (P - I) = \$\,45,500 - \$\,19,264.52 = \$\,26,235.48$.
   The remining balance after the partial payment is \$26,235.48.

2. The principal is \$150,000, which will be treated as the balance, B, of the loan. The annual simple interest rate, in decimal form, is 0.0585. The time is $t = 70$ days.
   Step 1: Determine the amount of the partial payment that is applied to interest. To find this, substitute the values above into the formula $I = P \times \frac{r}{365} \times t$ and calculate. Calculating, the amount of the payment that is applied to interest is $I = 150,000 \times \frac{0.0585}{365} \times 70 = 1,682.8767$. Rounding up, we have $I = \$1,682.88$.
   Step 2: The amount of the payment that is to be applied to the balance of the loan is partial payment minus the amount of the partial payment that is applied to the interest. The payment is \$50,000. The amount that is applied to the balance is $P - I = \$\,50,000 - \$\,1,682.88 = \$\,48,317.12$.
   Step 3: The remaining balance is found by subtracting the amount applied to the balance from the previous balance, or $B - (P - I) = \$\,150,000 - \$\,48,317.12 = \$\,101,682.88$.
   The remining balance after the partial payment is \$101,682.88.

Loan Payoff

Finally, we will determine the amount to be paid at the end of the loan. To do so, we apply the formula for the loan payoff to the remaining balance. However, the length of time for that remaining balance is the time between the partial payment and the day the loan is paid off.

Step 1: Determine the remaining balance after the partial payment.

Step 2: Calculate the number of days between the partial payment and the date the loan is paid off. This will be the time t in the payment formula.

Step 3: Calculate the amount to be paid at the end of the loan, or the payoff amount, using $\text{Payoff} = P + P \times \frac{r}{365} \times t$, where P is the remaining balance and t is the time found in Step 2.

Example 9 **Finding Loan Pay Off After a Partial Payment**

Laura takes out an \$18,400 loan for 120 days at 17.9% simple interest. She makes a partial payment of \$7,500 after 45 days. What is her payoff amount at the end of the loan?

Solution

The initial balance, or principal, of her loan is \$18,400. The interest rate in decimal form is 0.179. Her partial payment of \$7,500 is made after 45 days. Using these values, we can determine how much of the partial payment is applied to the balance. From there, we can determine her final loan payoff after 120 days.

Step 1: Determine the remaining balance after the partial payment. Using the partial payment process outlined in the previous example, we first find that the amount of the partial payment that is applied to the balance. Their interest paid in the partial payment is $I = P \times \frac{r}{365} \times t = 18,400 \times \frac{0.179}{365} \times 45 = 406.0603$, or \$406.07 (remember to round up!). Using this and that the loan amount was for \$18,400, the remaining balance on the loan after the partial payment is $B - (P - I) = \$\,18,400 - (\$\,7,500 - \$\,406.07) = \$\,11,306.07$.

Step 2: The number of days between the partial payment and the date that the loan is to be paid off is 120 – 45 = 75. This means that the time between the partial payment and the final payment is 75 days.

Step 3: To calculate the payoff amount, use $\text{payoff} = P + P \times \frac{r}{365} \times t$, with $P = \$11{,}306.07$ (the remaining balance), $t = 75$ (from Step 2) and $r = 0.179$. The payoff amount, then, is $\text{payoff} = 11,306.07 + 11,306.07 \times \frac{0.179}{365} \times 75 = 11,721.9165$. Rounding up, the payoff amount is \$11,721.92.

Repeated Partial Payments

Car loans and mortgages (loans for homes) are paid off through repeated partial payments, most often monthly payments. Since car loans are often 3 to 6 years, and mortgages 15 to 30 years, calculating each individual monthly payment one at a time is time consuming and tedious. Even a 3-year loan would involve applying the above steps 36 times! Fortunately, there is a formula for determining the amount of each partial payment for monthly payments on a simple interest loan.

FORMULA

The amount of monthly payments, A, for a loan with principal P, monthly simple interest rate r (in decimal form), for t number of months is found using the formula $A = P \times \frac{r\times(1+r)^t}{(1+r)^t-1}$. The monthly interest rate is the annual rate divided by 12. The number of months is the number of years times 12.

Example 10 **Calculating Car Payments**

Desiree buys a new car, by taking a loan out from her credit union. The balance of her loan is \$27,845.00. The annual interest rate that Desiree will pay is 7.3%. She plans to pay this off over 4 years. How much will Desiree's monthly payment be?

Solution

To use the formula for monthly payments, we need the principal, the interest rate, and the number of years. The principal is \$27,845. The annual rate, in decimal form, is 0.073. Dividing 0.073 by 12 gives the monthly interest rate $\frac{0.073}{12} = 0.00608\overline{3}$. She takes the loan out for 4 years, which is $t = 12 \times 4 = 48$ months. Substituting these values into the formula, $A = P \times \frac{r \times (1+r)^t}{(1+r)^t - 1}$, we calculate:

$$\begin{aligned}
A = 27,845 \times \frac{0.00608\overline{3} \times \left(1 + 0.00608\overline{3}\right)^{48}}{\left(1 + 0.00608\overline{3}\right)^{48} - 1} \\
= \quad 27,845 \times \frac{0.00608\overline{3} \times \left(1.00608\overline{3}\right)^{48}}{\left(1.00608\overline{3}\right)^{48} - 1} \\
= \quad 27,845 \times \frac{0.00608\overline{3} \times 1.337918996}{1.337918996 - 1} \\
= \quad 27,845 \times \frac{0.008139007}{0.337918996} \\
= \quad 27,845 \times 0.024085675 \\
= \quad 670.6656
\end{aligned}$$

Using the formula and rounding up to the next cent, we see that Desiree's monthly payment will be \$670.67.

TECH CHECK

The calculation of payments is long, and involves many steps. However, most spreadsheet programs, including Google Sheets, have a payment function. In Google Sheets, that function is PMT. To find the payment for an installment loan (like for a car), you need to enter the interest rate per period, the number of payments, and the loan amount. From Your Turn 6.36, the rate was 0.0476/12, the number of payments was 60, and the loan amount was \$23,660. In Google Sheets, select any cell and enter the following:

=PMT(0.0476/12,60,23660)

And click the enter key. Immediately, in the cell you selected, the payment of \$443.90 appears, though with a negative sign. The negative sign indicates it is a payment out of an account. Since we want to know the payment amount, we ignore the negative sign. The result, with the formula in the formula bar, is shown.

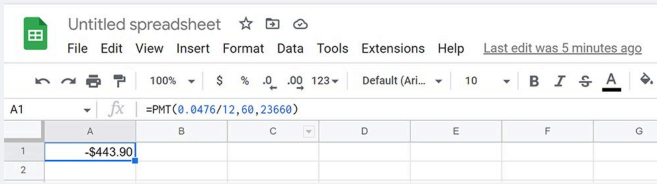


Figure 5: Google Sheets payment function

In general, to use the PMT function in Google Sheets, enter

=PMT(r/12,t*12,P)

where r is the annual interest rate, t is the number of years, and P is the principal of the loan.

Understand and Compute Present Value for Simple Interest Investments

When finding the future value of an investment, we know how much is deposited, but we have no idea how much that money will be worth in the future. If we set a goal for the future, it would be useful to know how much to deposit now so an account reaches the goal. The amount that needs to be deposited now to hit a goal in the future is called the **present value**.

FORMULA

The present value, PV, of money deposited at an annual, simple interest rate of r (in decimal form) for time t (in years) with a specified future value of FV, is calculated with the formula $PV = \frac{FV}{(1+rt)}$.

Note: Present value, in this calculation, is always rounded up. Otherwise, future value may fall short of the target future value.

Understanding what this tells you is important. When you find the present value, that is how much you need to invest now to reach the goal FV, under the conditions (time and rate) at which the money will be invested.

Example 11

Compute the present value of the investment described. Interpret the result.

1. $FV = \$10{,}000$, $t = 15$ years, annual simple interest rate of 5.5%
2. $FV = \$150{,}000$, $t = 20$ years, annual simple interest rate of 6.25%
3. $FV = \$250{,}000$, $t = 486$ months, annual simple interest rate of 4.75%

Solution

1. The future value is $FV = \$10{,}000$. The time of the investment is in years, so $t = 15$. The annual, simple interest rate is 5.5%, which in decimal form is 0.055. We substitute those values into the formula and calculate. $PV = \frac{FV}{(1+rt)} = \frac{10{,}000}{(1+(0.055)\times(15))} = \frac{10{,}000}{(1+0.825)} = \frac{10{,}000}{(1.825)} = 5{,}479.452$. Rounding up, we see that the present value of \$10,000 invested at a simple annual interest rate of 5.5% for 15 years is \$5,479.46. This means

that $5,479.46 needs to be invested so that, after 15 years at 5.5% interest, the investment will be worth $10,000.

2. The future value is $FV = \$150{,}000$. The time of the investment is in years, so $t = 20$. The annual, simple interest rate is 6.25%, which in decimal form is 0.0625. We substitute those values into the formula and calculate. $PV = \frac{FV}{(1+rt)} = \frac{150{,}000}{(1+(0.0625)\times(20))} = \frac{150{,}000}{(1+1.25)} = \frac{150{,}000}{(2.25)} = 66{,}666.\overline{6}$. Rounding up, we see that the present value of $150,000 invested at a simple annual interest rate of 6.25% for 20 years is $66,666.67. This means that $66,666.67 needs to be invested so that, after 20 years at 6.25% interest, the investment will be worth $150,000.

3. The future value is $FV = \$250{,}000$. The time of the investment is 486 months. This needs to be converted to years. To do so, divide the number of months by 12, giving $years = \frac{486}{12} = 40.5$, so $t = 40.5$ years. The annual, simple interest rate is 4.75%, which in decimal form is 0.0475. We substitute those values into the formula and calculate. $PV = \frac{FV}{(1+rt)} = \frac{250{,}000}{(1+(0.0475)\times(40.5))} = \frac{150{,}000}{(1+1.92375)} = \frac{150{,}000}{(2.92375)} = 129{,}954.5159$. Rounding up, we see that the present value of $250,000 invested at a simple annual interest rate of 4.75% for 486 months is $129,954.52. This means that $129,954.52 needs to be invested so that, after 486 months at 4.75% interest, the investment will be worth $150,000.

Example 12 **Present Value of a CD**

Beatriz will invest some money in a CD that yields 3.99% simple interest when invested for 30 years. How much must Beatriz invest so that after those 30 years, her CD is worth $300,000?

Solution

Beatriz needs to know how much to deposit now so that her CD is worth $300,000 after 30 years. This means she needs to know the present value of that $300,000. The time is 30 years and the annual simple interest rate, in decimal form, is 0.0399. Using that information and the formula for present value, we calculate the present value of that $300,000. $PV = \frac{FV}{(1+rt)} = \frac{300{,}000}{(1+(0.0399)\times(30))} = \frac{300{,}000}{(1+1.197)} = \frac{300{,}000}{(2.197)} = 136{,}549.8407$. Rounding up, Beatriz needs to invest $136,549.85 so that she has $300,000 in 30 years.

Key Terms

- Interest
- Principal
- Annual percentage rate
- Simple interest
- Term

- Due
- Origination date
- Payoff amount
- Future value
- Partial payment
- Present value

Key Concepts

- Interest is money that is paid by a borrower for the privilege of borrowing the money.
- Simple interest is computed by substituting the principal, interest rate, and number of years into the formula $I = P \times r \times t$
- The payoff for a loan is the amount of principal remaining on a loan plus the interest that accumulated on the loan since the last payment.
- The future value of an investment yielding simple interest is the original principal plus the interest earned on the investment.
- When making a partial payment, some of the payment pays off all the accumulated interest, while the remainder of the payment is applied to the principal of the loan.
- Finding the present value of an investment is used to determine how much should be invested now in order to achieve a specific goal.

Formulas

$$I = P \times r \times t$$

$$T = P + I$$

$$T = P + P \times r \times t$$

$$I = P \times \frac{r}{365} \times t$$

$$FV = P + I = P + P \times r \times t$$

$$A = P \times \frac{r \times (1 + r)^t}{(1 + r)^t - 1}$$

$$PV = \frac{FV}{(1 + rt)}$$

6.4 Compound Interest

Figure 6: The impact of compound interest

Learning Objectives

After completing this section, you should be able to:

1. Compute compound interest.
2. Determine the difference in interest between simple and compound calculations.
3. Understand and compute future value.
4. Compute present value.
5. Compute and interpret effective annual yield.

For a very long time in certain parts of the world, interest was not charged due to religious dictates. Once this restriction was relaxed, loans that earned interest became possible. Initially, such loans had short terms, so only simple interest was applied to the loan. However, when loans began to stretch out for years, it was natural to add the interest at the end of each year, and add the interest to the principal of the loan. After another year, the interest was calculated on the initial principal plus the interest from year 1, or, the interest earned interest. Each year, more interest was added to the money owed, and that interest continued to earn interest.

Since the amount in the account grows each year, more money earns interest, increasing the account faster. This growth follows a geometric series (Geometric Sequences). It is this feature that gives compound interest its power. This module covers the mathematics of compound interest.

Understand and Compute Compound Interest

As we saw in Simple Interest, an account that pays simple interest only pays based on the original principal and the term of the loan. Accounts offering **compound interest** pay interest at regular intervals. After each interval, the interest is added to the original principal. Later, interest is calculated on the original principal plus the interest that has been added previously.

After each period, the interest on the account is computed, then added to the account. Then, after the next period, when interest is computed, it is computed based on the original principal AND the interest that was added in the previous periods.

The following example illustrates how compounded interest works.

Example 1 **Interest Compounded Annually**

Abena invests \$1,000 in a CD (certificate of deposit) earning 4% compounded annually. How much will Abena's CD be worth after 3 years?

Solution

Since the interest is compounded annually, the interest will be computed at the end of each year and added to the CD's value. The interest at the end of the following year will be based on the value found form the previous year.

Step 1: After the first year, the interest in Abena's CD is computed using the interest formula $I = P \times r \times t$. The principal is $P = 1{,}000$, the rate, as a decimal, is 0.04, and the time is one year, so $t = 1$. Using that, the interest earned in the first year is $I = P \times r \times t = 1,000 \times 0.04 \times 1 = 40$, so the interest earned in the first year was \$40.00. This is added to the value of the CD, making the CD worth $\$\,1,000 + \$\,40 = \$\,1,040$.

Step 2: At the end of the second year, interest is again computed, but is computed based on the CD's new value, \$1,040. Using this new value and the interest formula (r and t are still 0.04 and 1, respectively), we see that the CD earned $I = P \times r \times t = 1,040 \times 0.04 \times 1 = 41.6$, or \$41.60. This is added to the value of the CD, making the CD now worth $\$\,1,040.00 + \$\,41.60 = \$\,1,081.60$.

Step 3: At the end of the third year, interest is again computed, but is computed based on Abena's CD's new value, \$1,081.60. Using this value and the interest formula (r and t are still 0.04 and 1, respectively), we see that the CD earned $I = P \times r \times t = 1,081.60 \times 0.04 \times 1 = 43.264$, or \$43.26 (remember to round down). This is added to the value of the CD, making the CD now worth $\$\,1,081.60 + \$\,43.26 = \$\,1,124.86$.

After 3 years, Abena's CD is worth \$1,124.86.

Determine the Difference in Interest Between Simple and Compound Calculations

It is natural to ask, does compound interest make much of a difference? To find out, we revisit Abena's CD.

Example 2 **Comparing Simple to Compound Interest on a 3-Year CD**

Abena invested $1,000 in a CD that earned 4% compounded annually, and the CD was worth $1,124.86 after 3 years. Had Abena invested in a CD with simple interest, how much would the CD have been worth after 3 years? How much more did Abena earn using compound interest?

Solution

Had Abena invested $1,000 in a 4% simple interest CD for 3 years, her CD would have been worth $P + P \times r \times t = 1,000 + 1,000 \times 0.04 \times 3 = 1,120$, or $1,120.00. With interest compounded annually, Abena's CD was worth $1,124.86. The difference between compound and simple interest is $\$\,1,124.86 - \$\,1,120.00 = \$\,4.86$. So compound interest earned Abena $4.86 more than the simple interest did.

VIDEO

Compound Interest

Understand and Compute Future Value

Imagine investing for 30 years and compounding the interest every month. Using the method above, there would be 360 periods to calculate interest for. This is not a reasonable approach. Fortunately, there is a formula for finding the future value of an investment that earns compound interest.

FORMULA

The future value of an investment, A, when the principal P is invested at an annual interest rate of r (in decimal form), compounded n times per year, for t years, is found using the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$. This is also referred to as the future value of the investment.

CHECKPOINT

Note, sometimes the formula is presented with the total number of periods, n, and the interest rate per period, r. In that case the formula becomes $A = P(1 + r)^n$.

Example 3 **Computing Future Value for Compound Interest**

In the following, compute the future value of the investment with the given conditions.

1. Principal is $5,000, annual interest rate is 3.8%, compounded monthly, for 5 years.
2. Principal is $18,500, annual interest rate is 6.25%, compounded quarterly, for 17 years.

Solution

1. The principal is $P = \$5,000$, interest rate, in decimal form, $r = 0.038$, compounded monthly so $n = 12$, and for $t = 5$ years. Substituting these values into the formula, we find

$$
\begin{aligned}
A = P\left(1+\frac{r}{n}\right)^{nt} &= 5,000\left(1+\frac{0.038}{12}\right)^{12\times 5} \\
= & \quad 5,000\left(1+0.0031\overline{6}\right)^{60} \\
= & \quad 5,000\left(1.0031\overline{6}\right)^{40} \\
= & \quad 5,000 \times 1.20888663572 \\
= & \quad 6,044.4332
\end{aligned}
$$

The future value of the investment is \$6,044.43.

1. The principal is P = \$18,500, interest rate, in decimal form, $r = 0.0625$, compounded quarterly so $n = 4$, and for $t = 17$ years. Substituting these values into the formula, we

$$
\begin{aligned}
A = P\left(1+\frac{r}{n}\right)^{nt} &= 18,500\left(1+\frac{0.0625}{4}\right)^{4\times 17} \\
= & \quad 18,500(1+0.015625)^{68} \\
= & \quad 18,500(1.015625)^{68} \\
= & \quad 18,500 \times 2.86992151999 \\
= & \quad 53,093.5481
\end{aligned}
$$

The future value of the investment is \$53,093.54.

Example 4 **Interest Compounded Quarterly**

Cody invests \$7,500 in an account that earns 4.5% interest compounded quarterly (4 times per year). Determine the value of Cody's investment after 10 years.

Solution

Cody's initial investment is \$7,500, so P = \$7,500. The annual interest rate is 4.5%, which is 0.045 in decimal form. Compounding quarterly means there are four periods in a year, so $n = 4$. He invests the money for 10 years. Substituting those values into the formula, we calculate

$$
\begin{aligned}
A = P\left(1+\frac{r}{n}\right)^{nt} &= 7,500\left(1+\frac{0.045}{4}\right)^{4\times 10} \\
= & \quad 7,500(1+0.01125)^{40} \\
= & \quad 7,500(1.01125)^{40} \\
= & \quad 7,500 \times 1.564376865391 \\
= & \quad 11,732.8265
\end{aligned}
$$

After 10 years, Cody's initial investment of \$7,500 is worth \$11,732.82.

Example 5 **Interest Compounded Daily**

Kathy invests $10,000 in an account that yields 5.6% compounded daily. How much money will be in her account after 20 years?

Solution

Kathy's initial investment is $10,000, so $P = \$10{,}000$. The annual interest rate is 5.6%, which is 0.056 in decimal form. Compounding daily means there are 364 periods in a year, so $n = 365$. She invests the money for 20 years, so $t = 20$. Substituting those values into the formula, we calculate

$$\begin{aligned} A = P\left(1+\frac{r}{n}\right)^{nt} &= 10,000\left(1+\frac{0.056}{365}\right)^{365\times 20} \\ &= 10,000(1+0.000153424657534)^{7300} \\ &= 10,000(1.000153424657534)^{7300} \\ &= 10,000 \times 3.06459091598 \\ &= 30,645.909 \end{aligned}$$

After 20 years, Kathy's initial investment of $10,000 is worth $30,645.90.

VIDEO

- Compare Simple Interest to Interest Compounded Annually
- Compare Simple Interest and Compound Interest for Different Number of Periods Per Year

WORK IT OUT

To truly grasp how compound interest works over a long period of time, create a table comparing simple interest to compound interest, with different numbers of periods per year, for many years would be useful. In this situation, the principal is $10,000, and the annual interest rate is 6%.

1. Create a table with five columns. Label the first column YEARS, the second column SIMPLE INTEREST, the third column COMPOUND ANNUALLY, the fourth column COMPOUND MONTHLY and the last column COMPOUND DAILY, as shown below.

YEARS	SIMPLE INTEREST	COMPOUND ANNUALLY	COMPOUND MONTHLY	COMPOUND DAILY

1. In the years column, enter 1, 2, 3, 5, 10, 20, and 30 for the rows.
2. Calculate the account value for each column and each year.

3. Compare the results from each of the values you find. How do the number of periods per year (compoundings per year) impact the account value? How does the number of years impact the account value?
4. Redo the chart, with an interest rate you choose and a principal you choose. Are the patterns identified earlier still present?

Understand and Compute Present Value

When investing, there is often a goal to reach, such as "after 20 years, I'd like the account to be worth $100,000." The question to be answered in this case is "How much money must be invested now to reach the goal?" As with simple interest, this is referred to as the present value.

FORMULA

The money invested in an account bearing an annual interest rate of r (in decimal form), compounded n times per year for t years, is called the present value, PV, of the account (or of the money) and found using the formula $PV = \frac{A}{\left(1+\frac{r}{n}\right)^{n\times t}}$, where A is the value of the account at the investment's end. Always round this value up to the nearest penny.

Example 6 **Computing Present Value**

Find the present value of the accounts under the following conditions.

1. $A = \$250{,}000$, invested at 6.75 interest, compounded monthly, for 30 years.
2. $A = \$500{,}000$, invested at 7.1% interest, compounded quarterly, for 40 years.

Solution

1. To reach a final account value of $A = \$250{,}000$, invested at 6.75% interest, in decimal form $r = 0.0675$ (decimal form!), compounded monthly, so $n = 12$, for 30 years, substitute those values into the formula for present value. Calculating, we find the present value of the $250,000.

$$\begin{aligned} PV = \frac{A}{\left(1+\frac{r}{n}\right)^{n\times t}} &= \frac{250{,}000}{\left(1+\frac{0.0675}{12}\right)^{12\times 30}} \\ &= \frac{250{,}000}{(1+0.005625)^{360}} \\ &= \frac{250{,}000}{(1.005625)^{360}} \\ &= \frac{250{,}000}{7.5332454772} \\ &= 33{,}186.2277 \end{aligned}$$

In order for this account to reach \$250,000 after 30 years, \$33,186.23 needs to be invested.

1. To reach a final account value of $A = \$500{,}000$, invested at 7.1% interest, in decimal form $r = 0.071$, compounded quarterly, so $n = 4$, for 40 years, substitute those values into the formula for present value. Calculating, we find the present value of the \$500,000.

$$\begin{aligned} PV = \frac{A}{\left(1+\frac{r}{n}\right)^{n\times t}} = & \frac{500,000}{\left(1+\frac{0.071}{4}\right)^{4\times 40}} \\ = & \frac{500,000}{(1+0.01775)^{160}} \\ = & \frac{500,000}{(1.01775)^{160}} \\ = & \frac{500,000}{16.6946672846} \\ = & 29,949.6834 \end{aligned}$$

In order for this account to reach \$500,000 after 40 years, \$29,949.69 needs to be invested.

Example 7 **Investment Goal with Compound Interest**

Pilar plans early for retirement, believing she will need \$1,500,000 to live comfortably after the age of 67. How much will she need to deposit at age 23 in an account bearing 6.35% annual interest compounded monthly?

Solution

Knowing how much to deposit at age 23 to reach a certain value later is a present value question. The target value for Pilar is \$1,500,000. The interest rate is 6.35%, which in decimal form is 0.0635. Compounded monthly means $n = 12$. She's 23 and will leave the money in the account until the age of 67, which is 44 years, making $t = 44$. Using this information and substituting in the formula for present value, we calculate

$$\begin{aligned} PV = \frac{A}{\left(1+\frac{r}{n}\right)^{n\times t}} &= \frac{1{,}500{,}000}{\left(1+\frac{0.0635}{12}\right)^{12\times 44}} \\ &= \frac{1{,}500{,}000}{\left(1+0.0052916\overline{6}\right)^{528}} \\ &= \frac{1{,}500{,}000}{\left(1.0052916\overline{6}\right)^{528}} \\ &= \frac{1{,}500{,}000}{16.226302189} \\ &= 92,442.5037 \end{aligned}$$

Pilar will need to invest \$92,442,51 in this account to have \$1,500,000 at age 67.

Compute and Interpret Effective Annual Yield

As we've seen, quarterly compounding pays interest 4 times a year or every 3 months; monthly compounding pays 12 times a year; daily compounding pays interest every day, and so on. **Effective annual yield** allows direct comparisons between simple interest and compound interest by converting compound interest to its equivalent simple interest rate. We can even directly compare different compound interest situations. This gives information that can be used to identify the best investment from a yield perspective.

Using a formula, we can interpret compound interest as simple interest. The effective annual yield formula stems from the compound interest formula and is based on an investment of \$1 for 1 year.

NOTE

Effective annual yield is $Y = \left(1+\frac{r}{n}\right)^n - 1$ where Y = effective annual yield, r = interest rate in decimal form, and n = number of times the interest is compounded in a year. Y is interpreted as the equivalent annual simple interest rate.

Example 8 Determine and Interpret Effective Annual Yield for 6% Compounded Quarterly

Suppose you have an investment paying a rate of 6% compounded quarterly. Determine and interpret that effective annual yield of the investment.

Solution

Here, $n = 4$ (quarterly) and $r = 0.06$ (decimal form). Substituting into the formula we find that the effective annual yield is

$$
\begin{aligned}
Y &= \left(1+\frac{0.06}{4}\right)^4 - 1 \\
&= (1.015)^4 - 1 \\
&= 1.06136 - 1 \\
&= 0.0614 \\
&= 6.14\%
\end{aligned}
$$

Therefore, a rate of 6% compounded quarterly is equivalent to a simple interest rate of 6.14%.

Example 9 **Determine and Interpret Effective Annual Yield for 5% Compounded Daily**

Calculate and interpret the effective annual yield on a deposit earning interest at a rate of 5% compounded daily.

Solution

In this case, the rate is $r = 0.05$ and $n = 365$ (daily). Using the formula $Y = \left(1+\frac{r}{n}\right)^n - 1$, we have

$$
\begin{aligned}
Y &= \left(1+\frac{0.05}{365}\right)^{365} - 1 \\
&= (1.0001369863)^{365} - 1 \\
&= 1.051267 - 1 \\
&= 0.0513
\end{aligned}
$$

This tells us that an account earning 5% compounded daily is equivalent to earning 5.13% as simple interest.

Example 10 **Choosing a Bank**

Minh has a choice of banks in which he will open a savings account. He will deposit \$3,200 and he wants to get the best interest he can. The banks advertise as follows:

Bank	Interest Rate
ABC Bank	2.08% compounded monthly
123 Bank	2.09% compounded annually
XYZ Bank	2.05% compounded daily

Which bank offers the best interest?

Solution

To compare these directly, Minh could change each interest rate to its effective annual yield, which would allow direct comparison between the rates. Computing the effective annual yield for all three choices gives:

ABC Bank: $Y = \left(1 + \frac{0.0208}{12}\right)^{12} - 1 = 0.0210 = 2.10\%$

123 Bank: $Y = \left(1 + \frac{0.0209}{1}\right)^{1} - 1 = 0.0209 = 2.09\%$

XYZ Bank: $Y = \left(1 + \frac{0.0205}{365}\right)^{365} - 1 = 0.0207 = 2.07\%$

ABC Bank has the highest effective annual yield, so Minh should choose ABC bank.

Key Terms

- Compound interest
- Effective annual yield

Key Concepts

- Compound interest means that the interest earned during one period will earn interest in later periods. Essentially, the amount of the principal grows from period to period.
- The important values in computing compound interest are the interest rate, the principal, the length of time the investment, and the number of times the investment is compounded.
- Compound interest has minimal impact early, but later has a very large impact.
- You can determine how much to invest today in order to reach a goal for some time later.
- Compound interest can be translated into an effective annual yield, which allows for comparison between investment options.

Videos

- Compound Interest
- Compare Simple Interest to Interest Compounded Annually
- Compare Simple Interest and Compound Interest for Different Number of Periods Per Year

Formulas

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$PV = \frac{A}{\left(1 + \frac{r}{n}\right)^{n\times t}}$$

$$Y = \left(1 + \frac{r}{n}\right)^{n} - 1$$

6.5 Making a Personal Budget

Figure 7: Calculating a budget is important to your financial health.

Learning Objectives

After completing this section, you should be able to:

1. Create a personal budget with the categories of expenses and income.
2. Apply general guidelines for a budget.

"That doesn't fit in the budget."

"We didn't budget for that."

"We need to figure out our budget and stick to it."

A **budget** is an outline of how money and resources should be spent. Companies have them, individuals have them, your college has one. But do you have one?

Creating a realistic budget is an important step in careful stewardship of your financial health. Designing your budget will help understand the financial priorities you have, and the constraints on your life choices. You want to have enough income to pay not only for the necessities, but also for things that represent your wants, like trips or dinner out. You also may want to save money for large purchases or retirement. You do not want to just get by, and you do not want the problems associated with overdue balances, rising debt, and possibly losing something you have worked hard to obtain.

While creating a budget may seem intimidating at first, coming up with your basic budget outline is the hardest part. Over time, you will adjust not only the numbers, but the categories.

Creating a Budget

You should view creating a **budget** as a financial tool that will help you achieve your long-term goals. A budget is an estimation of income and expenses over some period of time. You will be able to track your progress, which will help you to prepare for the future by making smart investment decisions.

There are several budget-creating tools available, such as the apps Good budget and Mint, and Google Sheets. Getting started, though, begins well before you find an app. The following are steps that can be used to create your monthly budget.

1. **Track your income and expenses** Review your income and expenses for the past 6 months to a year. This will give you an idea of your current habits.
2. **Set your income baseline** Determine all the sources of income you will have. This income may from paychecks, investments, or freelance work. It even includes child support and gifts. Be sure to use income after taxes. This allows you to determine your maximum expenditures per month.

> **CHECKPOINT**
> *For income that is not steady, such as gig work or freelance work, use the previous 6 to 12 months of income to find an average income from that gig or freelance work. Use this average in the budgeting process.*

1. **Determine your expenses** Review your bills from the past 6 months. You should include mortgage payments or rent, insurance, car payments, utilities, groceries, transportation expenses, personal care, entertainment, and savings. Using your credit card statements and bank statements will help you determine these amounts. Be aware that some of the expenses will not change over time. These are referred to as **fixed expenses**, like rent, car payments, insurance, internet service, and the like. Other expenses may vary widely from month to month and are appropriately called **variable expenses**, and include such expenses as gasoline, groceries.

> **CHECKPOINT**
> *Some expenses are yearly, such as insurance or property taxes. Other expanses may be quarterly (four times per year) or semiannual (twice per year). To budget for such bills by month, divide the bill total by the number of months the bill covers.*

1. **Categorize your expenses** These categories may be housing, transportation, or food, for broad categories, or may get more specific, where you categorize car payments, car insurance, and gasoline separately. The categories are your choices. Be sure to account for the cost of maintaining a vehicle or home. The more specific you are, the better you'll understand your spending needs and habits.

3. **Total your monthly income and monthly expenses and compare** These values should be compared. If your expenses are higher than your income, then adjustments have to be made. Decisions of what to do with any extra income is part of the planning process also.
4. **Make plans for unplanned expenses** Ask anyone, an unexpected car repair can ruin a carefully crafted budget. Have a plan for how you can be ready for these random expenses. This often means creating a cushion in your budget.

5. **Use your budget to make decisions and adjust for any changes** Your budget is a changeable document. Add to it when you wish, refer to it when special purchases are to be made. Keeping your budget up to date helps accommodate changes in income and expenses.

VIDEO

Creating a Budget

In this section, we will focus on income and expenses. One of the easiest ways to manage a budget is to create a table, with one column containing income sources, another with income values, a third with expense categories, and a last containing expenses. An example is shown.

Income Source	Amount	Expense	Amount
Full-time job	$3,565	Rent	$975
Uber	$185	Car Payment	$355
TOTAL	$3,750	Student Loan	$418
		Electric	$76
		Food	$400
		Gasoline	$250
		Car Insurance	$165
		Clothing	$100
		Entertainment	$100
		TOTAL	$2,839

WHO KNEW?

Gross Pay and Take Home Pay

If you've ever had a paycheck, you know that taxes are taken out of your pay before you get your check. This amount of money varies from state to state, and sometimes even city to city. For a person making $50,000 per year gross salary in Salt Lake City, Utah, take home pay is about 75.6% of gross salary. In Detroit, Michigan, take home pay is about 74.5% of gross salary. Lakeland, Florida, take home pay is about 80.5% of gross salary. These also change based on how much a person earns! Before choosing a place to live, it makes sense to determine how much deductions from pay will impact your income.

Example 1 **Creating a Budget**

Heather has graduated college and currently works as a nurse for a rural medical group. Her net monthly income from that job is $3,765.40. She also works part-time on the weekends, earning another $672.00 per month. Her monthly expenses are rent at $1,050, car payments at $489, student loan payments at $728, car insurance at $139, utilities at $130, clothing at $150, entertainment (going out with friends, Netflix, Amazon Prime, movies) at $300, credit card debt at $200, food at $360, and gasoline at $275. Create her budget in a table, compare the total income to total expenses, and determine how much excess income per month she has or how much she falls short by each month.

Solution

Step 1: To begin, we create the table with appropriate headings.

Income Source	Amount	Expense	Amount

Step 2: Her income categories are her nursing job, with $3,765.40 per month, and her part-time job, with $672.00 per month. Entering these into the table, we have the following.

Income Source	Amount	Expense	Amount
Nursing	$3,765.40		
Part-time	$672.00		

Step 3: Her monthly expenses are listed above. Entering the categories and the amount for each of those expenses, the table is now

Income Source	Amount	Expense	Amount
Nursing	$3,765.40	Rent	$1,050
Part-time	$672.00	Car Payment	$489
		Student Loan	$728
		Car Insurance	$139
		Utilities	$130
		Clothing	$150
		Entertainment	$300
		Credit Card	$200
		Food	$400
		Gasoline	$250

Step 4: Totaling the income and expenses, we see that her total income is \$4,437.40 per month, and her total expenses are \$3,836 per month. Comparing these, we see that Heather has \$601.40 in excess income per month. This provides a cushion in her budget.

Example 2 **Creating a Budget**

Carol is working in a dental lab, creating dentures and bridges. Monthly her take home pay is \$2,816 (based on \$22 per hour minus payroll taxes). She also receives \$320 per month in child support for her one daughter. Her monthly expenses are rent at \$700, car payments at \$229, student loan payments at \$250, car insurance at \$119, health insurance at \$225, utilities at \$80, clothing at \$75, entertainment at \$200, food at \$275, and gasoline at \$275. Create Carol's budget in a table, compare the total income to total expenses, and determine how much excess income per month she has or how much she falls short by each month.

Solution

Step 1: To begin, we create the table with appropriate headings.

Income Source	Amount	Expense	Amount

Step 2: Her income categories are from work, \$2,816, and child support, \$320, per month. Entering these into the table, we have the following.

Income Source	Amount	Expense	Amount
Job	\$2,816.00		
Child support	\$320.00		

Step 3: Her monthly expenses are listed above. Entering the categories and the amount for each of those expenses, the table is now

Income Source	Amount	Expense	Amount
Job	$2,816.00	Rent	$700
Child support	$320.00	Car Payment	$229
		Student Loan	$250
		Car Insurance	$119
		Utilities	$80
		Health insurance	$225
		Clothing	$75
		Entertainment	$200
		Food	$275
		Gasoline	$275

Step 4: Totaling the income and expenses, we see that her total income is $3,136.00 per month, and her total expenses are $2,428.00 per month. Comparing these, we see that Carol has $708.00 in excess income per month. This is the cushion in her budget.

Using the budget process, we can make decisions on adding expenses to the budget. To do so, check the cushion of the budget to see if there is room in the budget for the new expense.

Example 3 **Adding to an Existing Budget**

In the example above, Carol had excess income of $708.00. She looks up the cost of before-school care for her daughter. She finds that, monthly, the cost would be $252.00 per month. Is this an affordable program for Carol? Add this expense to her budget table.

Solution

She can afford this, as the cost for the before school program is $252.00 and she had extra income of $708.00. Adding this to her budget, her budget table is now

Income Source	Amount	Expense	Amount
Job	$2,816.00	Rent	$700
Child support	$320.00	Car Payment	$229
		Student Loan	$250
		Car Insurance	$119
		Utilities	$80
		Health insurance	$225
		Clothing	$75
		Entertainment	$200
		Food	$275
		Gasoline	$275
		Before-school care	$252

Now, she has $456.00 in excess income per month.

The 50-30-20 Budget Philosophy

It isn't clear, obvious, or easy to decide how much of your income to allocate to various categories of expenses. Many people pay their bills and then consider all the leftover money to be spending money. However, when developing your own budget, you may want to follow the **50-30-20 budget philosophy**, which provides a basic guideline for how your income could be allocated. Fifty percent of your budget is allotted to your needs, 30% of your budget is allotted to pay for your wants, and 20% of your budget is allotted for savings and debt service (paying off your debts).

Knowing what expenses are necessary and what expenses are wants is important, since wants and needs are often confused. The following are **necessary expenses** that represent basic living requirements and debt services. This list isn't complete: mortgage/rent, utilities, car, car insurance, health care, groceries, gasoline, child care (for working parents), and minimum debt payments. The 50-30-20 budget philosophy suggests that 50%, or half, your income go to these necessities.

Wants, though, are things you could live without but still wish to have, such as Amazon Prime, restaurant dinners, coffee from Starbucks, vacation trips, and hobby costs. Even a gym membership or that new laptop are wants. Creating the room to afford these wants is important to our mental health. Not budgeting for things we want will negatively impact our quality of life.

The remaining 20% should be set aside, either in retirement funds, stocks, other investments, an emergency fund (recommendations are that an emergency fund have 3 months of income), and perhaps extra spent to pay down debt. This 20% is very useful for addressing those unexpected costs, such as repairs or replacement of items that no longer work. Without budgeting this cushion, any expense that is a surprise can cause us to miss necessary payments.

The list of necessary expenses was not complete. There are other expenses that could be included.

Necessary Expenses and Expenses that are Wants

For some people, an expense will be necessary while the same expense for someone else will be a want. A good example of this is internet service. Many people consider internet service as a need, especially those who work from home or who are not able to leave their homes. One could also call internet service a need if they have children in school. For others, internet service is a want. If a person's job doesn't require them to be online, if they are not in school, if they do not have kids, then internet service can be dropped. There are public options for internet service. One could even use their phone as a hot spot.

Cars often fall into the category of need, but could also fall into the want category, depending on where and how you live. Bikes, public transportation, and walking are all options that could replace a car. This would then remove the cost of gasoline and car insurance.

Another consideration when deciding if an item on your budget is a need or a want is about your choices and priorities. A car is a need for many. But the need for a car is not the same as the need for a specific car. If you choose to buy a car with payments that exceed your budgeted amount for the car, then that car is a want. The amount you exceed the budget now belongs in the want category.

The same can be said for housing. If you want an apartment that costs $1,250 per month, but your budget only allows for an apartment that costs $900, then $350 of the rent is a want.

The point of that is to carefully consider if an expense is a need as opposed to a want.

When your expenses exceed your income, you may want to change how you budget your income to line up with these guidelines. This may mean cutting back, finding less-expensive living arrangements, finding a less-expensive (and more fuel-efficient) car, or sacrificing some specialty groceries. Using these guidelines keeps your financial life manageable.

Better still, they can guide you as you begin your life after graduation.

Example 4 **Evaluate a Budget Using 50-30-20 model**

In the example above, after Carol added before school care for her daughter to the budget, her budget was as shown below. Evaluate Carol's budget using the 50-30-20 budget philosophy.

Income Source	Amount	Expense	Amount
Job	$2,816.00	Rent	$700
Child support	$320.00	Car Payment	$229
		Student Loan	$250
		Car Insurance	$119
		Utilities	$80
		Health insurance	$225
		Clothing	$75
		Entertainment	$200
		Food	$275
		Gasoline	$275
		Before-school care	$252

Solution

Carol's total income is $3,136.00. Applying the 50-30-20 budget philosophy to this income requires the calculation of each of those percentages.

For the necessities, Carol should budget 50% of her income, or $0.5 \times \$\,3,136.00 = \$\,1,568.00$.

For her wants, she should budget 30% of her income, or $0.3 \times \$\,3,136.00 = \$\,940.80$.

For savings and extra debt service, she should budget 20% of her income, or $0.2 \times \$\,3,136.00 = \$\,627.20$.

In her budget, her necessities include all expenses except for entertainment. These expenses total $2,480, which exceeds the suggested budget amount of $1568.00. To follow the guidelines, Carol would have to cut back on these necessities.

For her wants, she spends $200.00 on entertainment, which is well below the suggested budget amount of $940.80. If she modifies how much she spends on needs, she may be able to increase the spending on her wants.

Her excess income is $456.00, which is below what she should be saving and using to pay down extra debt. If she does adjust how much she spends on needs, she could increase the amount for savings.

Example 5 **Creating a Budget Based on the 50-30-20 Budget Philosophy**

Carmen is about to graduate and has been offered a job at a bank as a data scientist. She estimates her monthly take home pay to be $5,662.50. Apply the 50-30-20 philosophy to that monthly income. How should Carmen use this information?

Solution

Step 1. To apply the 50-30-20 budget philosophy to Carmen's income, she needs to calculate 50%, 30%, and 20% of her income. Fifty percent of her income is $0.5 \times \$\,5,662.50 = \$\,2,831.25$. Thirty percent of her income is $0.3 \times \$\,5,662.50 = \$\,1,698.75$. Twenty percent of her income is $0.2 \times \$\,5,662.50 = \$\,1,132.50$.

Step 2. She would then budget $2,831.25 for her needs, $1,698.75 for her wants, and $1,132.50 for savings and debt service.

Step 3. When choosing where to live, what to eat, and what to drive, she should make choices that keep those costs, combined with her debt service costs, gasoline, and utilities, below $2,831.25. This means she will have to make decisions about what her priorities are.

Step 4. She should then figure out what she wants to do with her money, and stay within the limits, that is, keep those costs below $1,698.75.

Step 5. Finally, she can begin building her savings with the remaining $1,132.50.

Example 6 **Using the 50-30-20 Budget Philosophy to Analyze Affordability**

Steve is thinking of moving out of his family's home. He currently works at a full-time job making $18 per hour, which will give him, approximately, a net annual income of $29,180 (working 40 hours per week for 52 weeks per year). He has student debt that he pays off at $218.00 per month, and already owns a car that he pays $162.00 per month for.

1. Apply the 50-30-20 budget philosophy to Steve's income.
2. If he follows the budget, how much does he have, after paying his car payment and student loan, to spend on necessities.
3. If he follows the budget, how much will he set aside for wants? For savings?
4. Discuss the affordability of moving out, based on Steve's budget.

Solution

Before the 50-30-20 philosophy can be applied, Steve's monthly income needs to be determined. This is found by dividing his annual income by 12. This gives $\$\,29,\frac{180}{12} = \$\,2,431.67$. This will be used for his monthly budget.

1. To apply the 50-30-20 philosophy to Steve's income, find 50%, 30%, and 20% of his monthly income.

Needs (50%): 50% of his income is $0.50 \times \$\,2,431.67 = \$\,1,215.83$.
Wants (30%): $0.30 \times \$\,2,431.67 = \$\,729.50$
Savings (20%): $0.20 \times \$\,2,431.67 = \$\,486.33$

1. The total for Steve's needs is $1,215.83. From this, he already pays $218.00 for his student loans, and $162.00 for his car payment. Together that is $380.00. Subtracting from the amount he should budget for his needs, he can spend $835.83 on other needs.
2. Steve budgeted $729.50 for wants, and $486.33 for savings and other debt servicing.

3. Steve will have other needs to pay for, including rent, utilities, food, heath care, gasoline, and car insurance. It is difficult to imagine Steve being able to afford to move out, unless he reallocates money that he would want to save, or use for entertainment and other wants, or takes on another job. Even if Steve uses all the money that the 50-30-20 budget sets aside for savings, he still only has $1,322.16 to spend on those necessities. It does not appear he can afford to move out.

VIDEO

50-30–20 Budget Philosophy

Key Terms

- Budget
- Necessary expenses
- Fixed expenses
- Variable expenses
- 50-30-20 budget philosophy

Key Concepts

- A budget is a set of guidelines for how to allocate your income.
- Budgeting helps to plan for many of life's expenses
- Budgets are used to compare income to expenses. When expenses exceed income, changes have to be made.
- Budgets can help evaluate the affordability of life changes.
- One guideline for setting a budget is the 50-30-20 budget philosophy. The guidelines suggest that 50% of income is allocated to necessary expenses, 30% to expenses that wants, and 20% to savings and other debt reduction.

Videos

- Creating a Budget
- 50-30–20 Budget Philosophy

6.6 Methods of Savings

Figure 8: Money wisely invested grows over time.

Learning Objectives

After completing this section, you should be able to:

1. Distinguish various basic forms of savings plans.
2. Compute return on investment for basic forms of savings plans.
3. Compute payment to reach a financial goal.

The stock market crash of 1929 led to the Great Depression, a decade-long global downturn in productivity and employment. A state of shock swept through the United States; the damage to people's lives was immeasurable. Americans no longer trusted established financial institutions. By October 1931, the banking industry's biggest challenge was restoring confidence to the American public. In the next 10 years, the federal government would impose strict regulations and guidelines on the financial industry. The Emergency Banking Act of 1933 created the Federal Deposit Insurance Corporation (FDIC), which insures bank deposits. The new federal guidelines helped ease suspicions among the general public about the banking industry. Gradually, things returned to normal, and today we have more investment instruments, many insured through the FDIC, than ever before.

In this section, we will first look at the different types of savings accounts and proceed to discuss the various types of investments. There is some overlap, but we will try to differentiate among these financial instruments. Saving money should be a goal of every adult, but it can also be a difficult goal to attain.

Distinguish Various Basic Forms of Savings Plans

There are at least three types of savings accounts. Traditional savings accounts, certificates of deposit (CDs), and money market accounts are three main savings account vehicles.

Savings Account

A savings account is probably the most well-known type of investment, and for many people it is their first experience with a bank. A **savings account** is a deposit account, held at a bank or other financial institution, which bears some interest on the deposited money. Savings accounts are intended as a place to save money for emergencies or to achieve short-term goals. They typically pay a low interest rate, but there is virtually no risk involved, and they are insured by the FDIC for up to $250,000.

Savings accounts have some strengths. They are highly flexible. Generally, there are no limitations on the number of withdrawals allowed and no limit on how much you can deposit. It is not unusual, however, that a savings account will have a minimum balance in order for the bank to pay maintenance costs. If your account should dip below the minimum, there are usually fees attached.

WHO KNEW?

Many banks are covered by FDIC insurance. The FDIC is the Federal Deposit Insurance Corporation and is an independent agency created by the U.S. Congress. One of its purposes is to provide insurance for deposits in banks, including savings accounts. Be aware, not all banks are FDIC insured. The FDIC insures up to $250,000 for a savings account, so you do not want your balance to exceed that federally insured limit.

Having your savings account at the same bank as your checking account does offer a real advantage. For example, if your checking account is approaching its lower limit, you can transfer funds from your savings account and avoid any bank fees. Similarly, if you have an excess of funds in your checking account, you can transfer funds to your savings account and earn some interest. Checking accounts rarely pay interest.

PEOPLE IN MATHEMATICS

J.P. Morgan

J.P. Morgan was a wealthy banker around the turn of the 20th century. His business interests included railroads and the steel industry. However, it was in 1907 that a financial crisis, caused by poor banking decisions and followed by such great distrust in the banking system that a frenzy of withdrawals from banks occurred, that J.P. Morgan and other wealthy bankers lent from their own funds to help stabilize and save the system.

There are some weaknesses to savings accounts. Primarily, it is because savings accounts earn very low interest rates. This means they are not the best way to grow your money. Experts, though, recommend keeping a savings account balance to cover 3 to 6 months of living expenses in case you should lose your job, have a sudden medical expense, or other emergency.

Around tax time, you will receive a 1099-INT form stating the amount of interest earned on your savings, which is the amount that must be reported when you file your tax return. A **1099 form** is a tax form that reports earnings that do not come from your employer, including interest earned on savings accounts. These 1099 forms have the suffix INT to indicate that the income is interest income.

Savings accounts earn interest, and those earnings can be found using the interest formulas from previous sections. The final value of these accounts is sometimes called the future value of the account.

Example 1 **Single Deposit in a Savings Account**

Violet deposits $4,520.00 in a savings account bearing 1.45% interest compounded annually. If she does not add to or withdraw any of that money, how much will be in the account after 3 years?

Solution

To find the compound interest, use the formula from Compound Interest, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where A represents the amount in the account after t years, with initial deposit (or principal) of P, at an annual interest rate, in decimal form, of r, compounded n times per year. Violet has a principal of $4,520.00, which will earn an interest of $r = 0.0145$, compounded yearly (so $n = 1$), for $t = 3$ years. Substituting and calculating, we find that Violet's account will be worth

$$\begin{aligned} A = & \quad P\left(1 + \frac{r}{n}\right)^{nt} \\ & = \$\,4,520.00\left(1 + \frac{0.0145}{1}\right)^{1\times 3} \\ = & \quad \$\,4,520.00(1.0145)^3 \\ = & \quad \$\,4,719.48 \end{aligned}$$

Or, Violet will have $4,719.48 after 3 years.

WHO KNEW?

Banks have not always offered interest on savings accounts. An 1836 publication from Indiana noted that banks in other states allow small interest on deposits. It specifically says that in these other states, these deposits are what business transactions are based upon. And that giving interest would encourage deposits, and thus increase the business that banks can do.

Journal of the House of Representatives of the Sate of Indiana

Certificates of Deposit, or CDs

We discussed **certificates of deposit** (CDs) in earlier sections. CDs differ from savings accounts in a few ways. First, the investment lasts for a fixed period of time, agreed to when the money is invested in the CD. These time periods often range from 6 months to 5 years. Money from the CD cannot be withdrawn (without penalty) until the investment period is up. Also, money cannot be added to an existing CD.

Certificates of deposit have features similar to savings accounts. They are insured by the FDIC. They are entirely safe. They do, though, offer a better interest rate. The trade-off is that once the money is invested in a CD, that money is unavailable until the investment period ends.

Example 2 **5-Year CD**

Silvio deposits \$10,000 in a CD that yields 2.17% compounded semiannually for 5 years. How much is the CD worth after 5 years?

Solution

This also uses the compound interest formula from Compound Interest, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, Substituting the values $P = \$10{,}000$, $r = 0.0217$, $n = 2$ (semiannually means twice per year), and $t = 5$, we find the account will be worth

$$\begin{aligned} A = &\quad P\left(1 + \frac{r}{n}\right)^{nt} \\ = &\ \$\,10{,}000.00\left(1 + \frac{0.0217}{2}\right)^{2\times 5} \\ = &\quad \$\,10{,}000.00(1.01085)^{10} \\ = &\quad \$\,11{,}239.53 \end{aligned}$$

The CD will be worth \$11,219.53 after 5 years.

Money Market Account

A **money market account** is similar to a savings account, except the number of transactions (withdrawals and transfers) is generally limited to six each month. Money market accounts typically have a minimum balance that must be maintained. If the balance in the account drops below the minimum, there is likely to be a penalty. Money market accounts offer the flexibility of checks and ATM cards. Finally, the interest rate on a money market account is typically higher than the interest rate on a savings account.

Example 3 **Single Deposit to a Money Market Account**

Marietta opens a money market account, and deposits \$2,500.00 in the account. It bears 1.76% interest compounded monthly. If she makes no other transactions on the account, how much will be in the account after 4 years?

Solution

This, once again, uses the compound interest formula from Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$, Substituting the values $P = \$25{,}000$, $r = 0.0176$, $n = 12$, and $t = 4$, we find the account will be worth

$$\begin{aligned} A = &\quad P\left(1 + \frac{r}{n}\right)^{nt} \\ = &\ \$\,2{,}500.00\left(1 + \frac{0.0176}{12}\right)^{12\times 4} \\ = &\quad \$\,2{,}500.00\left(1.0014\overline{6}\right)^{48} \\ = &\quad \$\,2{,}682.20 \end{aligned}$$

The money market account will be worth \$2,682.20 after 4 years.

Return on Investment

If we want to compare the profitability of different investments, like savings accounts versus other investment tools, we need a measure that evens the playing field. Such a measure is **return on investment**.

FORMULA

The return on investment, often denoted ROI, is the percent difference between the initial investment, P, and the final value of the investment, FV, or ROI $= \frac{FV - P}{P}$, expressed as a percentage.

CHECKPOINT

The length of time of the investment is not considered in ROI.

Example 4 **Calculating Return on Investment**

1. Determine the return on investment for the 5-year CD from Example 2. Round the percentage to two decimal places.
2. Determine the return on investment for the money market account from Example 3. Round the percentage to two decimal places.

Solution

1. The initial deposit in the CD was \$10,000, so $P = \$10{,}000$. The value at the end of 5 years was \$11,239.53. so $FV = \$11{,}239.53$. Substituting and computing we find the return on investment.

$$\begin{aligned} \text{ROI} &= \frac{FV - P}{P} \\ &= \frac{\$\,11{,}239.53 - \$\,10{,}000}{\$\,10{,}000} \\ &= \frac{\$\,1{,}239.53}{\$\,10\,,\,000} \\ &= 0.123953 \end{aligned}$$

The ROI is 12.40%.

1. The initial deposit in the money market was \$2,500, so $P = \$2{,}500$. The value at the end of 4 years was \$2,682.20. so $FV = \$2{,}682.20$. Substituting and computing we find the return on investment.

$$\begin{aligned}\text{ROI} &= \frac{FV - P}{P} \\ &= \frac{\$\,2{,}682.20 - \$\,2{,}500}{\$\,2{,}500} \\ &= \frac{\$\,182.20}{\$\,2{,}500} \\ &= 0.07288\end{aligned}$$

The ROI is 7.29%.

VIDEO

Return on Investment, ROI

Annuities as Savings

In Compound Interest, we talked about the future value of a single deposit. In reality, people often open accounts that allow them to add deposits, or *payments*, to the account at regular intervals. This agrees with the 50-30-20 budget philosophy, where some income is saved every month. When a deposit is made at the end of each compounding period, such a savings account is called an **ordinary annuity**.

The formula for the future value of an ordinary annuity is $FV = pmt \times \frac{\left(1+\frac{r}{n}\right)^{n\times t} - 1}{\frac{r}{n}}$, where FV is the future value of the annuity, pmt is the payment, r is the annual interest rate (in decimal form), n is the number of compounding periods per year, and t is the number of years.

CHECKPOINT

It is important to note that the number of deposits per year and the number of periods per year are the same.

CHECKPOINT

Another form of annuity if the annuity due, which has deposits at the start of each compounding period. This other annuity type has different formulas and is not addressed in this text.

Example 5 **Future Value of an Ordinary Annuity**

Jill has an account that bears 3.75% interest compounded monthly. She decides to deposit \$250.00 each month, at the end of the compounding period, into this account. What is the future value of this account, after 8 years?

Solution

These are regular payments into an account bearing compound interest. She is depositing them at the end of each compounding period. This makes this an ordinary annuity. Substituting the values $pmt = 250$, $r = 0.0375$, $n = 12$, and $t = 8$ into the formula, we find the future value of the account.

$$\begin{aligned} FV = & \quad pmt \times \frac{\left(1+\frac{r}{n}\right)^{n\times t} - 1}{\frac{r}{n}} \\ = & \ 250 \times \frac{\left(1+\frac{0.0375}{12}\right)^{12\times 8} - 1}{\frac{0.0375}{12}} \\ = & \quad 250 \times \frac{(1.003125)^{96} - 1}{0.003125} \\ = & \ 250 \times \frac{1.34922752406 - 1}{0.003125} \\ = & \quad 250 \times \frac{0.34922752406}{0.003125} \\ = & \quad 250 \times 111.752807699 \\ = & \quad 27,938.202 \end{aligned}$$

The account, after 8 years, will contain $27,938.20.

WHO KNEW?

Setting Savings Account Interest Rates

There are a number of factors that contribute to the amount a bank gives for savings accounts. The interest rate reflects how much the bank values deposits. It also reflects the money that the bank will earn when they lend out money. Finally, interest rates are impacted by the Federal Reserve Bank. When the Fed raises interest rates, so do banks.

PEOPLE IN MATHEMATICS

The Federal Reserve Chairperson

The Federal Reserve Board monitors the risks in the financial system to help ensure a healthy economy for individuals, companies, and communities. The Board oversees the 12 regional reserve banks. The Chairperson of the Federal Reserve Board testifies to Congress twice per year, meets with the secretary of the Treasury, chairs the Federal Open Market Committee, and is the face of federal monetary policy. Currently, the Fed Chair is Jerome Powell, who has served since 2018.

Example 6 **Saving for College**

When Yusef was born, Rita and George began to save for Yusef's college years by investing $2,500 each year in a savings account bearing 3.4% interest compounded annually. How much will they have saved after 18 years?

Solution

To find the future value of the account, we use the ordinary annuity formula $FV = pmt \times \frac{\left(1+\frac{r}{n}\right)^{n\times t}-1}{\frac{r}{n}}$. The payment is $2,500, rate is 0.034, the number of compounding periods is 1, and the number of years is 18. Substituting these values and computing, we have

$$\begin{aligned} FV = & \quad pmt \times \frac{\left(1+\frac{r}{n}\right)^{n\times t}-1}{\frac{r}{n}} \\ = & \ \$\,2,500 \times \frac{\left(1+\frac{0.034}{1}\right)^{1\times 18}-1}{\frac{0.034}{1}} \\ = & \quad \$\,2,500 \times \frac{(1.034)^{18}-1}{0.034} \\ = & \ \$\,2,500 \times \frac{1.82544897331-1}{0.034} \\ = & \quad \$\,2,500 \times 24.2779109798 \\ = & \quad \$\,60,694.77 \end{aligned}$$

After saving for 18 years, Rita and George will have $60,694.77 for Yusef's college.

TECH CHECK

Google Sheets offers a function to calculate the future value of an ordinary annuity. To get Google Sheets to calculate the future value, you use the following:

=fv(rate,number_of_periods, payment, present_value, end_or_beginning).

To explain, the rate is the rate per compounding period. From our formula, that is $\frac{r}{n}$. Also, the number of periods must be entered. From our formula, that is $n \times t$. The payment is the amount deposited each period. Present value is 0 if we begin with no money and rely only on the payments to be made. However, if some money is available to put in the account before the payments start, that amount, an initial deposit, would be the value of PV. Finally, for an ordinary annuity, enter 0 for end or beginning. Using the values for Jill, the payment amount is $250, $r = 0.0375$, $n = 12$, $t = 8$, and that there is no initial deposit, $PV = 0$, the Google Sheets formula is

=fv(0.0375/12,12*8,250,0,0).

shows the formula in Google Sheets.

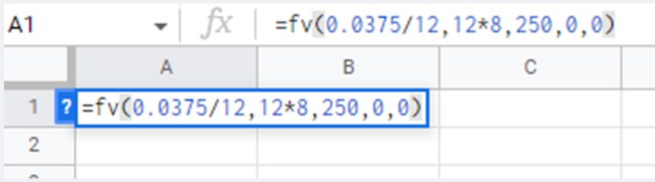


Figure 9: Google Sheets formula

Hitting the enter key shows the payment value .

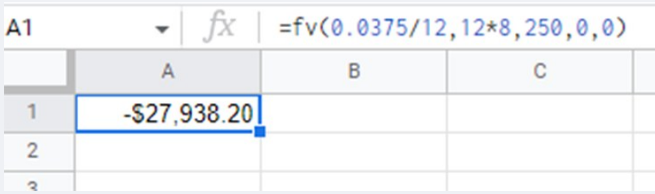


Figure 10: Payment value

Notice that the future value is negative, since it is a payment leaving an account.

VIDEO

Future Value Using Google Sheets

Compute Payment to Reach a Financial Goal

The formula used to get the future value of an ordinary annuity is useful, finding out what the final amount in the account will be. However, that isn't how planning works. To plan, we need to know how much to put into the ordinary annuity each compounding period in order to reach a goal. Fortunately, that formula exists.

FORMULA

The formula for the amount that needs to be deposited per period, *pmt*, of an ordinary annuity to reach a specified goal, *FV*, is $pmt = \dfrac{FV \times \left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{n\times t} - 1}$, where *r* is the annual interest rate (in decimal form), *n* is the number of periods per year, and *t* is the number of years.

With this formula, it is possible to plan the amount to be saved.

Example 7 **Saving for a Car**

Yaroslava wants to save in order to buy a car, in 3 years, without taking out a loan. She determines that she'll need $35,500 for the purchase. If she deposits money into an ordinary annuity that yields 4.25% interest compounded monthly, how much will she need to deposit each month?

Solution

Yaroslava has a goal and needs to know the payments to make to reach the goal. Her goal is $FV = \$35{,}500$, with an interest rate $r = 0.0425$, compounded per month so $n = 12$, and for 3 years, making $t = 3$. Substituting into the formula, Yaroslava finds the necessary payment.

$$\begin{aligned} pmt &= \frac{FV \times \left(\frac{r}{n}\right)}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1} \\ &= \frac{35{,}500 \times \left(\frac{0.0425}{12}\right)}{\left(1 + \frac{0.0425}{12}\right)^{12 \times 3} - 1} \\ &= \frac{35{,}500 \times \left(0.003541\overline{6}\right)}{\left(1.003541\overline{6}\right)^{36} - 1} \\ &= \frac{125.7291\overline{6}}{0.13572901696} \\ &= 926.325 \end{aligned}$$

To reach her goal, Yaroslava would need to deposit \$926.33 in her account each month.

CHECKPOINT

This has been rounded up, so that the deposits don't fall short of the goal. However, some round off using the standard rounding rules: if the last digit is 1, 2, 3, or 4, the number is rounded down; if the last digit is 5, 6, 7, 8, or 9 the number is rounded up.

TECH CHECK

Google Sheets offers a function to calculate the payment necessary to reach a goal using ordinary annuities. To get Google Sheets to calculate the payment, you use the following:

=pmt(rate,number_of_periods, present_value, future_value, end_or_beginning).

To explain, the rate is per compounding period. From our formula, that is $\frac{r}{n}$. Also, the number of periods must be entered. From our formula, that is $n \times t$. The present value is the amount of money that the account begins with. If we begin with no money and rely only on the payments to be made, then this number is 0. However, if some money is available to put in the account before the payments start, that amount, an initial deposit, would be the value of PV. Next, enter the future value, FV. Finally, for an ordinary annuity, enter 0 for end or beginning. Using the values for Yaroslava, $r = 0.0425$, $n = 12$, $t = 3$, and that there is no initial deposit, $PV = 0$, the Google Sheets formula is

=pmt(0.0425/12,12*3,0,35500,0).

shows the formula in Google Sheets.

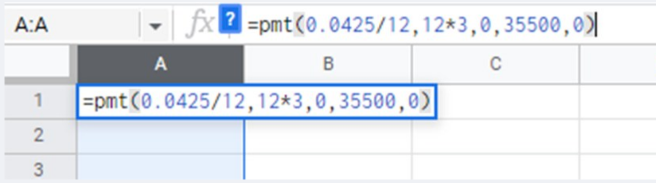


Figure 11: Google Sheets formula

Hitting the enter key shows the payment value .

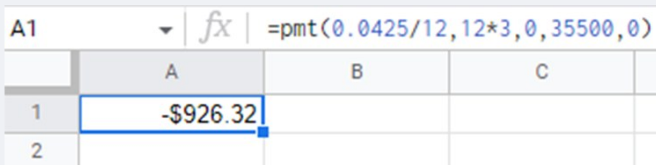


Figure 12: Payment value

Notice that the payment is negative, since it is a payment leaving an account. Additionally, the payment is $926.32. We rounded that up, but Google Sheets rounded off.

Key Terms

- Savings account
- 1099 form
- Certificate of deposit
- Money market account
- Return on investment
- Ordinary annuity

Key Concepts

- There are three main types of savings accounts, saving accounts, certificates of deposit (CD), and money market accounts.
- Savings account are very risk free, and so yield low interest rates.
- The differences in the three types of savings accounts relate to their convenience.
- Savings account typically have a lower interest rate that money market accounts, which typically have lower interest rates than CDs.
- Ordinary annuities more accurately reflect how we save, in that money is deposited repeatedly over time.
- Spreadsheet software, such as Google Sheets, have built in functions that can be used to quickly calculate both the future value of an ordinary annuity account, but also the payment necessary to reach a goal using an ordinary annuity.

Videos

- Return on Investment, ROI
- Future Value Using Google Sheets

Formulas

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\text{ROI} = \frac{FV - P}{P}$$

$$FV = pmt \times \frac{\left(1 + \frac{r}{n}\right)^{n \times t} - 1}{\frac{r}{n}}$$

$$pmt = \frac{FV \times \left(\frac{r}{n}\right)}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1}$$

6.7 Investments

Figure 13: Stocks are bought and sold to improve investment values.

Learning Objectives

After completing this section, you should be able to:

1. Distinguish between basic forms of investments including stocks, bonds, and mutual funds.
2. Understand what bonds are and how bond investments work.
3. Understand how stocks are purchased and gain or lose value.
4. Read and derive information from a stock table.
5. Define a mutual fund and how to invest.
6. Compute return on investment for basic forms of investments.
7. Compute future value of investments.

8. Compute payment to reach a financial goal.
9. Identify and distinguish between retirement savings accounts.

   You can save your money in a safe or a vault (or worse, under the mattress!), but that money does not grow. It would be hard to save enough for retirement that way. What can be done to increase the value of the money you already have?

   The answer is to invest it. Use the money that you have to earn more money back. For instance, as we saw in Methods of Savings, you can save it in a bank. Or, to reach loftier goals, invest in something more likely to grow, such as stocks.

   A great example of this is Apple stock. Anyone who bought stock in Apple Inc. (formerly Apple Computer, Inc.) in 1997 and held onto the shares earned a lot of money. To be more specific, \$100 worth of Apple shares bought in 1980, when it was first sold to the public, was valued at \$67,564 in 2019, or 676 times more! Perhaps you have heard a story like that, of an investment opportunity taken that paid off, or the story of an investment opportunity missed. But such stories are the exceptions.

   In this section, we'll investigate bonds, stocks, and mutual funds and their comparative strengths and weaknesses. We close the section with a discussion of retirement savings accounts.

Distinguish Between Basic Forms of Investments

Bonds, stocks, and mutual funds tend to offer higher returns, but to varying degrees, come with higher risks. Stocks and mutual funds also vary in how much they earn. Their predicted rates of return on investment are not guaranteed, but educated guesses based on market trends and historical performance.

We will use the methods and formulas we learned earlier to evaluate these forms of investment.

Bonds

Bonds are issued from big companies and from governments. Selling bonds is an alternative to an institution taking a loan from a bank. The funds from the selling of bonds are often used for large projects, like funding the building of a new highway or hospital.

Bonds are considered a conservative investment. They are bought for what is known as the **issue price**. The interest is fixed (does not change) at the time of purchase and is based on the issue price of the bond. The interest rate is often referred to as the coupon rate; the interest paid is often called the coupon yield. The interest paid is often higher than savings accounts and the risk is exceptionally low. The bond is for a fixed length of time. The end of this time is the maturity date of the bond.

There are several types of bonds:

- **Treasury bonds** are issued by the federal government.
- **Municipal bonds** are issued by state and local governments.
- **Corporate bonds** are issued by major corporations.

  There are other types of bonds available, but they are beyond the scope of this section.

WHO KNEW?

Trading Bonds

Bonds are often part of larger investment portfolios. These bonds may be traded. However, the interest paid is based on the price when the bond was bought (the issue price). These bonds can be bought and sold for more or less money than the issue price. If the bond is bought for more than the issue price, the interest is still paid on the issue price, not on the purchase price when the trade was made. This means the actual return on the bond decreases. If the bond is bought for less than the issue price, the return on the bond goes up.

VIDEO

Bonds

Example 1

Muriel purchases a $3,000 bond with a maturity of 4 years at a fixed coupon rate of 5.5% paid annually. How much is Muriel paid each year, and how much does she receive on the maturity date?

Solution

The coupon rate is 5.5%. 5.5% of her bond value is $0.055 \times \$\,3,000 = \$\,165$. After year 1, Muriel receives $165. She receives $165 after years 2 and 3 also. In year 4, when the bond matures, Muriel receives $3,165, or the interest and the initial investment, or principal.

Stocks

Stocks are part ownership in a company. They come in units called **shares**. The performance and earnings of stocks is not guaranteed, which makes them riskier than any other investment discussed earlier. However, they can offer higher return on investment than the other investments. Their value grows in two ways. They offer **dividends**, which is a portion of the profit made by the company. And the price per share can increase based on how others see that value of the company changing. If the value of the company drops, or the company folds, the money invested in the stock also drops.

Most stock transactions are executed through a broker. Brokers' commissions can be a percentage of value of the trades made or a flat fee. There are full-service brokers who charge higher commission rates, but they also offer financial advice and perform the research that you may not have the time or the expertise to do on your own. A discount broker only executes the stock transactions, buying or selling, so they charge lower rates than full-service brokers. There are also brokers that offer commission-free trading.

An important thing to remember is that stocks might provide a very large return on investment, but the trade-off is the risk associated with owning stocks.

WHO KNEW?

Chapter 11 Bankruptcy and Stocks

In the fall of 2022, the parent company of Regal Theaters, named Cineworld, filed for Chapter 11 bankruptcy. According to news articles, the bankruptcy was necessitated due to its heavy debt load. Generally, a company can file for Chapter 11 bankruptcy to allow them time to reorganize and restructure debts. When this happens, the company, after the Chapter 11 process is over, offers new stock. This makes the previous stock worthless. However, the company may allow an exchange of old stock for a discounted amount of the new stock. This in effect reduces (maybe vastly) the wealth held by those who owned the original stock.

Example 2 **Buying Stock in Company ABC**

Haniah buys stock in the ABC company, investing a total of $13,000. She expects the stock to grow, through stock price increase and reinvestment of dividends, by 12.3% per year and compounded annually. If she leaves that money invested, how much will the stocks be worth in 20 years?

Solution

Calculating this is a compound interest calculation, if Haniah's assumption about the stock's performance is correct. If so, then the principal is $13,000, the rate is 0.123, the number of compounding periods per year is 1, and the time is 20 years. Substituting into the compound interest formula from Methods of Savings, and computing, we have $A = P\left(1 + \frac{r}{n}\right)^{nt} = \$\,13,000\left(1 + \frac{0.123}{1}\right)^{1 \times 20} = \$\,13,000 \times 10.1764223996 = \$\,132,293.49$. After 20 years, her stock is now worth $132,293.49.

WHO KNEW?

Risk and Volkswagen

The question of risk hovers over every investment. How risky can it get? Volkswagen seems to be a rather safe investment. But in 2015, Volkswagen's stock tumbled 30% over a few days when it was revealed that the company had installed software that altered the emission performance of some of their diesel engines. Volkswagen's hope was that lower emissions would bolster US sales of some of their diesel models. This was a drastic drop, and many investors lost a lot of money. However, the stock has come back since then. This was mild compared to the 65% drop in the Martha Stewart Living Omnimedia stocks.

PEOPLE IN MATHEMATICS

Warren Buffett

Warren Buffett is an investment legend. He began his career as an investment salesman in the 1950s. He formed Buffett associates in 1956. In 1965, he was in control of Berkshire Hathaway, which began as a merger between two textile companies. In his role there, he began to invest in a variety of companies. It is now a conglomerate holding company, and fully owns GEICO, Duracell, Diary Queen, and other large companies.

His investment philosophy involves finding stocks and bonds from companies that have high intrinsic worth compared to their stock or bond prices. This means he focuses not on the supply and demand side of stock investing, but instead on the company's worth in total. Using this philosophy, he has become one of the world's most successful investors.

Reading Stock Tables

Information about particular stocks is contained in **stock tables**. This information includes how much the stock is selling for, and its high and low values form the past year (52 weeks). In a newspaper, the stock table may look like this:

52-Week High Low		Stock	SYM	Div	Yld %	P/E	Vol 100s	High	Low	Close	Net Chg
41.66	18.90	McDonalds	MCD	.72	2.9	12	7588	25.73	23.87	25.42	+0.31
22.60	13.20	Mon-santo	MON	.52	2.4	55	15474	21.86	21.48	21.64	−0.29
17.05	8.30	Motorola	MOT	.16	1.7	dd	16149	10.57	8.88	10.43	+0.14
31.75	22.99	Mueller	MLI	-	-	16	1564	29.32	27.03	27.11	−0.02

The symbols and abbreviations are defined here:

52-week High	52-week Low	The highest and lowest price of the stock over the past 52 weeks
Stock	SYM	The name of the company and the symbol used for trading
	Annual DIV	The current annual dividend per share
	Yld %	Percent yield is $= \frac{\text{annual dividend}}{\text{share price}} \times 100$
	P/E	Price to earnings ratio, share price divided by earnings per share over past year (dd indicates loss)
	Vol 100s	The number of shares traded yesterday in 100s
High	Low	The highest and lowest prices at which stocks traded yesterday
	Close	The price at which the stock traded at the close of the market yesterday
	Net Chg	Net change; change in price from market close 2 days ago to yesterday's close

The formulas for yield and price to earnings is a good way to measure how much the stock returns per share. Their values are calculated in the stock table, but deserve attention here.

FORMULA

The price to earnings ratio of a stock, P/E, is $\frac{\text{P}}{\text{E}} = \frac{\text{Share Price}}{\text{Dividend}}$. The percent yield for a stock, Yld%, is $\text{Y l d \%} = \frac{\text{Annual Dividend}}{\text{Share Price}} \times 100\%$.

It should be noted that the price of a stock increases and decreases every moment, and so these value change as the share price changes.

Example 3 **Computing Percent Yield**

1. Find the percent yield for a stock with a price of \$30.69 and an annual dividend of \$1.48.
2. Find the percent yield for a stock with a price of \$62.25 and an annual dividend of \$1.76.

Solution

1. Substituting the values for price, \$30.69, and annual dividend, \$1.48, we find the percent yield for the stock to be $\text{Y l d \%} = \frac{\text{Annual Dividend}}{\text{Share Price}} \times 100\% = \frac{\$\,1.48}{\$\,30.69} \times 100\% = 4.82\%$
2. Substituting the values for price, \$62.25, and annual dividend, \$1.76, we find the percent yield for the stock to be $\text{Yld\%} = \frac{\text{Annual Dividend}}{\text{Share Price}} \times 100\% = \frac{\$\,1.76}{\$\,62.25} \times 100\% = 2.83\%$

The stock table information is now, and has been, available online, from websites such as cnn.com/markets, markets.businessinsider.com/stocks, and marketwatch.com. The same information is available from these sites as from the newspaper listings, but are often accessed one stock at a time. shows the stock table for Lowe's on September 7, 2022.

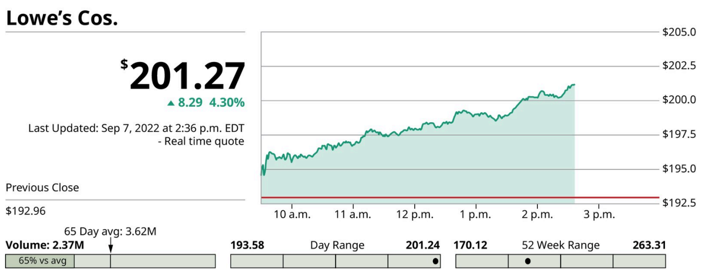


Figure 14: Key data for Lowe's stock 9/7/2022 (data source: marketwatch.com)

Other key data is further down on the website, and is shown in , below.

Key Data

Open **$193.71**	Day Range **193.58–201.44**
52 Week Range **170.12–263.31**	Market Cap **$119.77B**
Shares Outstanding **620.7M**	Public Float **620.17M**
Beta **1.14**	Rev. Per Employee **$280.57K**
P/E Ratio **15.79**	EPS **$12.69**
Yield **2.10%**	Dividend **$1.05**
Ex-Dividend Date **Oct 18, 2022**	Short Interest **11.09M** 08/15/22
% of Float Shorted **1.79%**	Average Volume **3.62M**

Performance

5 Day	3.74%
1 Month	−0.01%
3 Month	4.65%
YTD	−22.08%
1 Year	−1.32%

Figure 15: Key data for Lowe's stock 9/7/2022 (data source: marketwatch.com)

Notice that the 52-week high and low are now shown as the 52-week range. However, you get additional information, including the stock performance over the past 5 days, past month, past 3 months, the year to date (YTD), and over the past year. You can also read the number of shares outstanding, the expected date for the dividend (EX-DIVIDEND DATE), and importantly for the P/E ratio, the earning per share (EPS).

Example 4 **Reading an Online Stock Table**

Consider the stock table , and answer the questions based on the table.

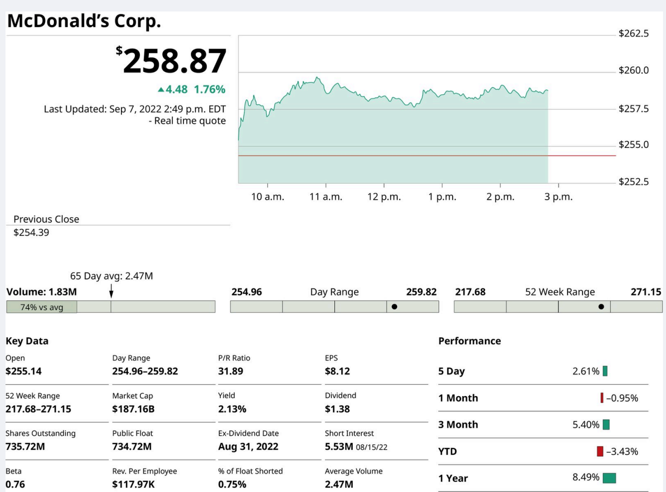


Figure 16: Key data for McDonald's stock 9/7/2022 (data source: marketwatch.com)

1. What is the current price for McDonald's Corp on this date?
2. What is the 52-wk high? 52-wk low?
3. When is the dividend expected?

4. What is its yield?
5. What is the earnings per share?

Solution

1. Looking at the table, the current price of a share is $258.87.
2. The high was $271.15, and the low was $217.68.
3. August 31, 2022
4. 2.13%
5. The EPS value is $8.12.

As mentioned, stocks earn money in two ways, through dividends and increase in share price.

Example 5 **Dividends Paid**

Darma owns 150 shares of stock in the GDW company. This quarter, GDW is paying $0.87 per share in dividends. How much will Darma earn in dividends this quarter?

Solution

Each share pays $0.87, so Darma earns $150 \times \$\,0.87 = \$\,130.50$.

Example 6 **Stock Price Increases**

Vincent buys 100 stocks in the REM company for $21.87 per share. One year later, he sells those 100 shares for $29.15 per share.

1. How much money did Vincent make?
2. What was his return on investment for that one year?

Solution

1. Vincent spent $21.87 per share to buy the stock. The total he spent on the stock was $\$\,21.87 \times 100 = \$\,2,187.00$. When he sold the stock, the price was $29.15, so he received $\$\,29.15 \times 100 = \$\,2,915.00$. He made $\$\,2,915.00 - \$\,2,187.00 = \$\,728.00$.
2. His return on investment was $\frac{\text{Earnings}}{\text{Original Price}} = \frac{\$\,728}{\$\,2{,}187} = 33.29\%$.

VIDEO

Reading Stock Summary Online

Mutual Funds

A **mutual fund** is a collection of investments that are all bundled together. When you buy shares of a mutual fund, your money is pooled with the assets of other investors. This pooled money is invested in stocks, bonds, money market instruments, and other assets. Mutual funds are typically operated by professional money managers who allocate the fund's assets and attempt to produce capital gains or income for the fund's investors.

A key benefit of mutual funds is that they allow small or individual investors to invest in professionally managed portfolios of equities, bonds, and other securities. This means each shareholder participates proportionally in the gains or losses of the fund. The performance of a mutual fund is usually stated as how much the mutual fund's total value has increased or decreased. Since there are many different investments inside the mutual fund, the risk is reduced significantly, compared to direct ownership of stocks. Even so, mutual funds historically perform well and can earn more than 10% annually.

The investments that make up a mutual fund are structured and maintained to match stated investment objectives, which are specified in its **prospectus**. A prospectus is a pamphlet or brochure that provides information about the mutual fund. Before buying shares of a mutual fund, consult its prospectus, consider its goals and strategies to see if they match your goals and values and also research any associated fees.

VIDEO

Mutual Funds

Example 7 **Investing in a Mutual Fund**

Kaitlyn has analyzed her \$12,862.50 quarterly budget using the 50-30-20 budget philosophy, and sees she should be saving or paying down debt with \$2,572.50 per quarter. She decides to invest \$1,300 quarterly a mutual fund that reports an average return of 11.62% over the 18-year life of the mutual fund. Assuming that this interest rate continues, and is compounded quarterly, how much will her mutual fund account be worth after 5 years?

Solution

Kaitlyn's plan is an ordinary annuity, and so the future value of her account can be found using the formula $FV = pmt \times \frac{\left(1+\frac{r}{n}\right)^{n \times t}-1}{\frac{r}{n}}$, with a payment of \$1,300, a rate of 0.1162, number of compounding periods 4, after 5 years. Substituting these values into the formula and calculating, we find

$$\begin{aligned} FV = & \quad pmt \times \frac{\left(1+\frac{r}{n}\right)^{n \times t}-1}{\frac{r}{n}} \\ = & \quad \$\,1,300 \times \frac{(1.02905)^{20}-1}{0.02905} \\ = & \; \$\,1,300 \times \frac{0.773084935178}{0.032905} \\ = & \quad \$\,1,300 \times 26.6122189784 \\ = & \quad \$\,34,595.88 \end{aligned}$$

Kaitlyn's mutual fund will be worth \$34,595.88 after 5 years.

Example 8 **Investing in a Mutual Fund to Reach a Goal**

Kaitlyn wants to retire with $1,500,000 in her mutual fund account. She will invest for 35 years. The mutual fund reports an average return of 11.62% over the 18-year-long life of the mutual fund. Assuming that this interest rate continues, and is compounded quarterly, how much will she need to pay annually into her mutual fund to reach her goal?

Solution

Kaitlyn's plan is an ordinary annuity, and so the payment to reach her goal can be found using the formula $pmt = \frac{FV \times \left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{n \times t} - 1}$, with a FV, or goal, of $1,500,000, a rate of 0.1162, for 35 years. Substituting these values into the formula and calculating, we find

$$\begin{aligned} pmt &= \frac{FV \times \left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{n \times t} - 1} \\ &= \frac{\$\ 1{,}500{,}000 \times \left(\frac{0.1162}{1}\right)}{\left(1+\frac{0.1162}{1}\right)^{1 \times 35} - 1} \\ &= \frac{\$\ 174{,}300}{26.0558103113} \\ &= \$\ 6,689.49 \end{aligned}$$

Kaitlyn needs to invest $6,689.49 per year (or $557.46 per month) into the mutual fund to reach $1,500,000 in 35 years.

Return on Investment

As in Methods of Savings, the formula for return on investment is $\text{ROI} = \frac{FV - P}{P}$. As indicated before, this formula does not take into account how long the investment took to reach its current value. It depends only on the initial value, P, and the value at the end of the investment, FV.

Example 9 **Return on Investment for a Bond**

Recall Example 1, in which Muriel purchased a $3,000 bond with a maturity of 4 years at a fixed coupon rate of 5.5% paid annually. What was Muriel's return on investment?

Solution

Each year, Muriel received $165. She received this money four times, so earned a total of $660. This represents $FV - P$, or just the earnings. Using that we find that the ROI is $\text{ROI} = \frac{660}{3000} = 0.22$, or 22%.

As mentioned, the ROI does not address the length of time of the investment. A good way to do that is to equate the ROI to an account bearing interest that is compounded annually.

The annual return is the average annual rate, or the annual percentage yield (APY) that would result in the same amount were the interest paid once a year.

FORMULA

The formula for annual return is annual return $= \left(\frac{FV}{P}\right)^{\left(\frac{1}{t}\right)} - 1$, where t = the number of years, FV = new value, and P = starting principal.

We apply this to the previous example.

Example 10 **Annual Return on Investment for a Bond**

Recall Example 9, in which Muriel purchased a $3,000 bond with a maturity of 4 years at a fixed coupon rate of 5.5% paid annually. What was Muriel's annual return on investment? Interpret this as compound interest.

Solution

Muriel earned a total of $660. This represents $FV - P$, or just the earnings. The starting principal was $3,000. The value at the end of 4 years was $3,000 + $660 = $3,660. The time of the investment was 4 years. Using that we find that the annual return is annual return $= \left(\frac{FV}{P}\right)^{\left(\frac{1}{t}\right)} - 1 = \left(\frac{\$\,3{,}660}{\$\,3{,}000}\right)^{\left(\frac{1}{4}\right)} - 1 = 1.22^{\frac{1}{4}} - 1 = 1.050969 = 0.050969$, or 5.10%. The 5.5% bond earned the equivalent of 5.10% compounded annually.

In Example 10 and Your Turn, the annual return was lower than the interest rate of the investment. This is because the interest from a bond is simple interest, but annual yield equates to compounded annually.

Example 11 **Return on Investment for Stock in Company ABC**

Haniah buys stock in the ABC company, investing a total of $13,000. After 20 years, the stock is worth $132,293.49, including reinvestment of dividends.

1. What is Haniah's return on investment?
2. What is Haniah's annual return?

   Solution

   1. To calculate Hanniah's return on investment, substitute $13,000 for P and $132,293.49 for FV in the formula ROI $= \frac{FV-P}{P}$ and calculate. Doing so we find Haniah's return on investment to be ROI $= \frac{FV-P}{P} = \frac{\$\,132{,}293.49 - \$\,13{,}000}{\$\,13{,}000} = 9.176422$, or 917.64%
   2. To calculate Haniah's annual return, substitute $13,000 for P and $132,293.49 for FV in the formula annual return $= \left(\frac{FV}{P}\right)^{\left(\frac{1}{t}\right)} - 1$ and calculate. Doing so we find her annual return to be annual return $= \left(\frac{FV}{P}\right)^{\left(\frac{1}{t}\right)} - 1 = \left(\frac{\$\,132{,}293.49}{\$\,13{,}000}\right)^{\left(\frac{1}{20}\right)} - 1 = 0.1230$, or 12.3%

You should see that the annual return is equal to the annual compounded interest that was assumed for the stocks.

Compute Payment to Reach a Financial Goal

As in Methods of Savings, determining the payment necessary to reach a financial goal uses the payment formula for an ordinary annuity, $pmt = \frac{FV \times \left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{n\times t}-1}$. If dealing with mutual funds or stocks, an assumed annual interest rate, compounded, will be used. This value is often determined through research and informed speculation.

Example 12

Richard is saving for new siding for his home. He and his partner believe they will need \$37,500 in 10 years to pay for the siding. How much should they invest yearly in a mutual fund they believe will have an annual interest rate of 12%, compounded annually, in order to reach their goal?

Solution

The necessary annual payment is found using the function $pmt = \frac{FV \times \left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{n\times t}-1}$ with $FV = 37{,}500$, $r = 0.12$, and $n = 1$. Substituting and calculating, we find the annual payment should be

$$\begin{aligned} pmt &= \frac{FV \times \left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{n\times t}-1} \\ &= \frac{\$\,37{,}500 \times \left(\frac{0.12}{1}\right)}{\left(1+\frac{0.12}{1}\right)^{1\times 10}-1} \\ &= \frac{\$\,4{,}500}{(1.12)^{10}-1} \\ &= \frac{\$\,4{,}500}{2.10584820834} \\ &= \$\,2,136.91 \end{aligned}$$

Retirement Savings Plans

We close this section by investigating the three main forms of retirement savings accounts: traditional individual retirement accounts (IRAs), Roth IRAs, and 401(k) accounts. Each has distinct characteristics that are suited to different investors' needs.

Individual Retirement Accounts

A **traditional IRA** lets you contribute up to an amount set by the government, which may change from year to year. For example, the maximum contribution for 2022 is \$6,000; \$7,000 over age 50. Anyone is eligible to contribute to a traditional IRA, regardless of your income level. Your money grows tax-deferred, but withdrawals after age 59½ are taxed at current rates.

Traditional IRAs also allow you to use the contribution itself as a deduction on a current year tax return.

Roth IRAs allow contributions at the same levels as traditional IRAs, with a maximum $6,000 for 2022; $7,000 over age 50. However, to be eligible to make contributions, your earned income must be below a certain level. A Roth IRA allows after-tax contributions. In other words, the contribution itself is not tax-deductible, as it is with the traditional IRA. However, your money grows tax-free. If you make no withdrawals until you are age 59½, there are no penalties. IRAs pay a modest interest rate.

In either case, IRA deposits have to be from earned income, which in effect means if your earned income is over $6,000 ($7,000) then you can deposit the maximum.

Example 13 **Comparing Roth IRAs to Traditional IRAs**

Which type of IRA, Roth or traditional, has an income limit for its use?

Solution

Roth IRAs require income to be below a certain limit.

WHO KNEW?

In 2022, the maximum that can be added to a Roth IRA was $6,000 for those under 50 years of age. For those over 50 years of age, the maximum that can be added to a Roth IRA is $7,000. However, to qualify for a Roth IRA in fall of 2022, a single person's modified adjusted gross income (MAGI) must be below $129,000. Then, if a single person's income is between $129,000 and $144,000, the maximum contribution is reduced from the limit for incomes below $129,000. For a married couples filing a joint tax return those values are $204,000 to $214,000.

401(k) Accounts

Your employer may offer a retirement account to you. These are often in the form of a **401(k)** account. There are traditional and Roth 401(k) accounts, which differ in how they are taxed, much as with other IRAs. In the traditional 401(k) plans, the money is deposited before tax is assessed, which means you do not pay taxes on this money. However, that means when money is withdrawn, it is taxed. These accounts are similar to mutual funds, in that the money is invested in a wide range of assets, spreading the risk.

One of the perks some employers offer is to match some amount of your contributions to the 401(k) plan. For instance, they may match your deposits up to 5% of your income. This is an instant 100% return on the money that was matched.

VIDEO

401(k) Accounts

Example 14 **Matching 401(k) Deposit**

Alice signs up for her employer-based 401(k). The employer matches any 401(k) contribution up to 6% of the employee salary. Alice's annual salary is $51,600.

1. What is the most money that Alice can deposit that will be fully matched by the company?
2. How much total will be deposited into Alice's account if she deposits the full 6%?
3. How much return does Alice earn if she deposits exactly 6% in her 401(k)?

Solution

1. The employer will match up to 6% of any employee's salary. 6% of Alice's salary is $0.06 \times \$\,51,600 = \$\,3,096$. So Alice can deposit up to $3,096 and receive that amount in matching funds in her account.
2. Alice's contribution plus the company's contribution is $\$\,3,096 + \$\,3,096 = \$\,6,192$, which is the total that is deposited into Alice's account.
3. She earns a 100% return on the day she deposits her $3,096.

401(k) plans with matching funds provide great value, as their rates of return are high compared to savings accounts, and are less risky that stocks since such funds invest across many investment vehicles. The next example demonstrates the power of constant deposits into a 401(k) plan that has some employer match.

Example 15 **Constant Deposits into a 401(k) Plan**

DeJean begins depositing $300 per month from his paycheck each month in his employer-based 401(k) account. The employer matches this deposit as it falls below their matching threshold. DeJean expects the return to average 10% per year, compounded annually.

1. How much will DeJean's account be worth if he keeps making those payments for 30 years?
2. What will his account be worth without the matching funds?

Solution

1. This is a form of an ordinary annuity, so the formula $FV = pmt \times \frac{\left(1+\frac{r}{n}\right)^{n \times t}-1}{\frac{r}{n}}$ will be used. The company matches DeJean's full deposit, so each month $600 will be deposited. He is assuming the money will compound annually, so the amount deposited each year is needed as the value of pmt. For the year, he will deposit $12 \times \$\,600 = \$\,7,200$. The rate is 0.1, the number of compounding periods is 1, and the number of years is 30. Substituting and calculating, the value of DeJean's account after 30 years will be

$$\begin{aligned} FV &= pmt \times \frac{\left(1+\frac{r}{n}\right)^{n\times t}-1}{\frac{r}{n}} \\ &= \$\,7,200 \times \frac{\left(1+\frac{0.1}{1}\right)^{1\times 30}-1}{\frac{0.1}{1}} \\ &= \$\,7,200 \times \frac{(1.1)^{30}-1}{0.1} \\ &= \$\,7,200 \times 164.494022689 \\ &= \$\,1,184,356.96 \end{aligned}$$

2. This is a form of an ordinary annuity, so the formula $FV = pmt \times \frac{\left(1+\frac{r}{n}\right)^{n\times t}-1}{\frac{r}{n}}$ will be used but the deposit is now only \$300 per month without the matching funds. For the year, he will deposit $12 \times \$\,300 = \$\,3,600$. The rate is 0.1, the number of compounding periods is 1, and the number of years is 30. Substituting and calculating, the value of DeJean's account after 30 years will be

$$\begin{aligned} FV &= pmt \times \frac{\left(1+\frac{r}{n}\right)^{n\times t}-1}{\frac{r}{n}} \\ &= \$\,3,600 \times \frac{\left(1+\frac{0.1}{1}\right)^{1\times 30}-1}{\frac{0.1}{1}} \\ &= \$\,3,600 \times \frac{(1.1)^{30}-1}{0.1} \\ &= \$\,3,600 \times 164.494022689 \\ &= \$\,592,178.48 \end{aligned}$$

Key Terms

- Bonds
- Maturity date
- Stocks
- Dividend
- Mutual fund
- Prospectus
- Issue price

- Shares
- Stock table
- Individual retirement account
- Roth IRA
- 401(k)

Key Concepts

- There are many different investments with different returns and risks.
- Bonds are loans form the purchaser to the entity selling the bond.
- Bonds have some tax benefits, low to no risk, and a low return.
- Stocks represent part ownership in a company. As such, stock holders share in the profits, and losses, of the company.
- Information, including price, P/E, yearly highs and lows, and dividend amount can be found in online stock tables available on many websites.
- Mutual funds represent collections of professionally administered investment vehicles. Have shares in a mutual fund has lower risk than ownership of stocks.
- Retirement accounts employ some of the same strategies as mutual funds, in that they spread the risk and are professionally managed.
- IRAs and Roth IRAs differ on when taxes are paid on the money, and who can use them. Roth IRAs have income limits while traditional IRAs do not.

Videos

- Bonds
- Reading Stock Summary Online
- Mutual Funds
- 401(k) Accounts

Formulas

$$\text{annual return} = \left(\frac{FV}{P}\right)^{\left(\frac{1}{t}\right)} - 1$$

$$\frac{\text{P}}{\text{E}} = \frac{\text{Share Price}}{\text{Dividend}}$$

$$\text{Yld }\% = \frac{\text{Annual Dividend}}{\text{Share Price}} \times 100\ \%$$

6.8 The Basics of Loans

Figure 17: Loans are contracts that allow people to buy now but require them to pay more.

Learning Objectives

After completing this section, you should be able to:

1. Describe various reasons for loans.
2. Describe the terminology associated with loans.
3. Understand how credit scoring works.
4. Calculate the payment necessary to pay off a loan.
5. Read an amortization table.
6. Determine the cost to finance for a loan.

New car envy is real. Some people look at a new car and feel that they too should have a new car. The search begins. They find the model they want, in the color they want, with the features they want, and then they look at the price. That's often the point where the new car fever breaks and the reality of borrowing money to purchase the car enters the picture. This borrowing takes the form of a loan.

In this section, we look at the basics of loans, including terminology, credit scores, payments, and the cost of borrowing money.

Reasons for Loans

Even if you want a new car because you need one, or if you need a new computer since your current one no longer runs as fast or smoothly as you would like, or you need a new chimney because the one on your house is crumbling, it's likely you do not have that cost in cash. Those

are very large purchases. How do you buy that if you don ’t have the cash?
You borrow the money.

And for helping you with your purchase, the company or bank charges you interest.

Loans are taken out to pay for goods or services when a person does not have the cash to pay for the goods or services. We are most familiar with loans for the big purchases in our lives, such as cars, homes, and a college education. Loans are also taken out to pay for repairs, smaller purchases, and home goods like furniture and computers.

Loans can come from a bank, or from the company selling the goods or providing the service. The borrower agrees to pay back more than the amount borrowed. So there is a cost to borrowing that should be considered when deciding on a purchase bought with credit or a borrowed money. Even using a credit card is a form of a loan.

Essentially, a loan can be obtained for just about any purchase, large or small, that has a cost beyond a person’s cash on hand.

The Terminology of Loans

There are many words and acronyms that get used in relation to loans. A few are below.

APR is the annual percentage rate. It is the annual interest paid on the money that was borrowed. The **principal** is the total amount of the loan, or that has been financed. A **fixed interest rate** loan has an interest rate that does not change during the life of the loan. A **variable interest rate** loan has an interest rate that may change during the life of the loan. The **term** of the loan is how long the borrower has to pay the loan back. An **installment loan** is a loan with a fixed period, and the borrower pays a fixed amount per period until the loan is paid off. The periods are almost uniformly monthly. **Loan amortization** is the process used to calculate how much of each payment will be applied to principal and how much is applied to interest. **Revolving credit**, also known as open-end credit, is how most credit cards work but is also a kind of loan account. (We will learn about credit cards in Credit Cards) You can use up to some specified value, called the limit, any way you want, and as long as you pay the issuer of the credit according to their terms, you can keep borrowing from this account.

These and other terminologies can be researched further at Forbes.

WHO KNEW?

Credit Scores

Not everyone pays the same interest for the same loan. One person might get an APR of 2.9% while another pays 6.9%. These rates are based on your credit score.

Data about you and your credit is collected by three credit bureaus—Experian, Equifax, and TransUnion. They calculate your score using one of two main models: FICO and Vantage Score. The score they develop is based on the following categories:

- **Payment History:** Making your payments on time and not missing payments is by far the most important factor. All three credit types—revolving, installment, and open—contribute to this factor.

- **Credit Utilization or Amount Owed:** How much do you owe on your credit card accounts? This category is concerned with the ratio of how much you owe on revolving credit accounts relative to your available credit, also known as your credit utilization ratio. This is the only category that depends solely on your revolving credit accounts.
- **Length of Credit History:** This is the average age of your credit history, including the age of the oldest and newest accounts. All three types of credit accounts play a role in this category.
- **Credit Mix:** This number represents the different types of credit accounts you have, such as credit cards, car loan, mortgages, and whether you are successful managing both revolving and installment accounts.
- **New Credit:** Have you recently opened a new account or applied for new credit? Lenders want to know how much new credit you are taking on. So, if you are planning to buy a car and make another large purchase with a credit card, you may want to space these purchases out.

If you have done well in these categories, your credit score will be high, and you will qualify for lower interest rates because you are not perceived as being a risky investment. However, if you do poorly in these categories, your score will be low and you will pay higher interest rates since you present a greater risk.

Check out this nerdwallet article on credit scores to learn more!

VIDEO

Credit Scores Explained

WHO KNEW?

Where Do Interest Rates Come From?

There are many factors that impact your interest rate beyond your credit score. Banks have the authority to set their own rates, so competition between banks impacts interest rates. Bank A doesn't want to charge interest rates that are too high, since borrowers will find banks with better rates. Banks also don't want to charge too little interest.

The too little interest is more involved than the too high. The bank needs to make a profit on its loans. Deposits at the bank are used by the bank to generate loans. The bank has to pay those depositors interest. The bank must charge more for loans they give than they pay to people with deposits in the bank.

Banks also borrow money from each other. These loans have an interest rate, and once more, the bank making a loan must make a profit. More directly, they must charge more for loans they give than they pay for loans they take. In the United States, banks may also borrow from the Federal Reserve, which also charges an interest rate, which is also called the discount rate.

> This is where a bank's prime rate comes from. A bank's prime rate is the interest rate it will give to its very best customers, which means most customers will pay more than the bank's prime rate. To confuse the issue, there is also the *Wall Street Journal*'s prime rate. It is the average of the prime rates charged by individual banks. The *Wall Street Journal* surveys several banks to generate this value.
>
> Banks also increase the interest rate charged to customers based on both the credit risk presented by the customer, and the risk associate with what the loan will be used for.

Calculating Loan Payments

Loan payments are made up of two components. One component is the interest that accrued during the payment period. The other component is part of the principal. This should remind you of partial payments from Simple Interest.

Over the course of the loan, the amount of principal remaining to be paid decreases. The interest you pay in a month is based on the remaining principal, just as in the partial payments of Simple Interest.

> **FORMULA**
>
> The amount of interest, I, to be paid for one period of a loan with remaining principal P is $I = P \times \frac{r}{n}$, where r is the interest rate in decimal form and n is he number of payments in a year (most often $n = 12$). Since the interest is for the one period, the time is 1 and does not impact the calculation. Note, interest paid to lenders is always rounded up to the next penny.

Example 1 **Interest for a Monthly Payment of a Loan**

Find the interest to be paid for the period on loans with the following remaining principal and given annual interest rate. Each period is a month.

1. Remaining principal is \$13,450, interest rate is 6.75%
2. Remaining principal is \$8,460, interest rate is 5.99%

Solution

1. Substituting \$13,450 for the remaining principal P, 0.0675 for r, and $n = 12$ since the period is a month into the formula, we find that the interest to be paid this period is $I = P \times \frac{r}{n} = \$\,13,450 \times \frac{0.0675}{12} = \$\,75.66$.
2. Substituting \$8,460 for the remaining principal P, 0.0599 for r, and $n = 12$ since the period is a month into the formula, we find that the interest to be paid this period is $I = P \times \frac{r}{n} = \$\,8,450 \times \frac{0.0599}{12} = \$\,42.18$.

The payment of the loan has to be such that the principal of the loan is paid off with the last payment. In any period, the amount of interest is defined by the formula above, but changes from period to period since the principal is decreasing with each payment. The trick is knowing how much principal should be paid each payment so that the loan is paid off at the stated time. Fortunately, that is found using the following formula.

FORMULA

The payment, pmt, per period to pay down a loan with beginning principal P is $pmt = \frac{P \times \left(\frac{r}{n}\right) \times \left(1+\frac{r}{n}\right)^{n \times t}}{\left(1+\frac{r}{n}\right)^{n \times t} - 1}$, where r is the annual interest rate in decimal form, t is the number of years of the loan, and n is the number of payments per year (typically, loans are paid monthly making $n = 12$).

Note, payment to lenders is always rounded up to the next penny.

CHECKPOINT

Often, the formula takes the form $pmt = \frac{P \times (r) \times (1+r)^n}{(1+r)^n - 1}$, *where r is the interest rate per period (annual rate divided by the number of periods per year), and n is the total number of payments to be made.*

Example 2 **Calculating the Payment for a Loan**

In the following, calculate the payment necessary to pay off the loan with the given details. The payments are monthly.

1. A car loan taken out for $28,500 at an annual interest rate of 3.99% for 5 years.
2. A home loan taken out for $136,700 and an annual interest rate of 5.75% for 15 years.

Solution

1. The loan is for $28,500, which is the principal. The rate is 3.99%, so $r = 0.0399$. The term of the loan is 5 years, so $t = 5$. Monthly payments means $n = 12$. Substituting these values for P, r, n, and t into the formula $pmt = \frac{P \times \left(\frac{r}{n}\right) \times \left(1+\frac{r}{n}\right)^{n \times t}}{\left(1+\frac{r}{n}\right)^{n \times t} - 1}$ and calculating, we find the payment for the loan.

$$\begin{aligned} pmt &= \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n\times t}}{\left(1 + \frac{r}{n}\right)^{n\times t} - 1} \\ &= \frac{\$\,28\,,\,500 \times \left(\frac{0.0399}{12}\right) \times \left(1 + \frac{0.399}{12}\right)^{12\times 5}}{\left(1 + \frac{0.0399}{12}\right)^{12\times 5} - 1} \\ &= \frac{\$\,28\,,\,500 \times (0.003325) \times (1.003325)^{60}}{(1.003325)^{60} - 1} \\ &= \frac{\$\,115.6470437}{0.220388273} \\ &= \$\,524.75 \end{aligned}$$

The monthly payment needed is $524.75.

2. The loan is for $136,000, which is the principal P. The rate is 5.75% so $r = 0.0575$. The term of the loan is 15 years, so $n = 15$. Monthly payments mean n =12. Substituting these values for P, r, n, and t into the formula $pmt = \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n\times t}}{\left(1 + \frac{r}{n}\right)^{n\times t} - 1}$ and calculating, we find the payment for the loan.

$$\begin{aligned} pmt &= \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n\times t}}{\left(1 + \frac{r}{n}\right)^{n\times t} - 1} \\ &= \frac{\$\,136\,,\,700 \times \left(\frac{0.0575}{12}\right) \times \left(1 + \frac{0.0575}{12}\right)^{12\times 15}}{\left(1 + \frac{0.0575}{12}\right)^{12\times 15} - 1} \\ &= \frac{\$\,28\,,\,500 \times (0.0047917) \times (1.0047917)^{180}}{(1.003325)^{180} - 1} \\ &= \frac{\$\,1\,,\,548.600986}{1.364201118} \\ &= \$\,1,135.18 \end{aligned}$$

The monthly payment needed is $1,135.18.

TECH CHECK

Using Google Sheets to Calculate Loan Payments

Google Sheets has a formula to calculate the monthly payment necessary to pay off a loan with a specified interest rate and term. The formula is the PMT formula. It uses the principal P, interest rate r, and t years. Follow these steps

Open a Google worksheet and click on any cell.

Type =-PMT(r/12,12*t,P).

Hit the enter key.

The cell displays the payment.

Please note the negative sign. Since Google Sheets is a spreadsheet program, is sees the payment as funds leaving the account, and so they are, by default, negative. The negative sign in front of the formula makes the result positive.

For example, for a $50,000 loan at 10.9% interest for 7 years, you would type =-PMT(0.109/12,12*7,50000). The formula and the result are shown.

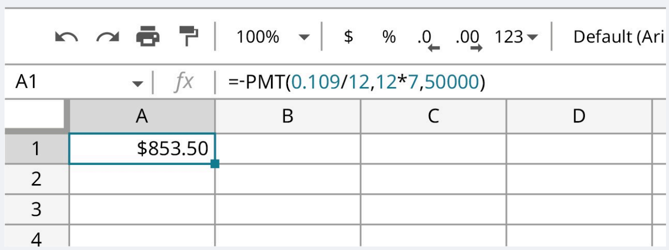


Figure 18: Google Sheets formula

Alternatively, you can also use an online calculator to find monthly payments for a loan, such as the one at Caluculator.net

Reading Amortization Tables

An **amortization table** or amortization schedule is a table that provides the details of the periodic payments for a loan where the payments are applied to both the principal and the interest. The principal of the loan is paid down over the life of the loan. Typically, the payments each period are equal. Importantly, one of the columns will show how much of each payment is used for interest, another column shows how much is applied to the outstanding principal, and another column shows the remaining principal or balance .

Amortization Schedule Calculator					
Loan Amount		$5,000.00			
Interest Rate (annual)		8.00%			
Term (Years)		5			
Monthly Payment		$101.38			
Month	Payment	Principal	Interest	Total Interest	Balance
					$5,000.00
1	$101.38	$68.05	$33.33	$33.33	$4,931.95
2	$101.38	$68.50	$32.88	$66.21	$4,863.45
3	$101.38	$68.96	$32.42	$98.64	$4,794.49
4	$101.38	$69.42	$31.96	$130.60	$4,725.07
5	$101.38	$69.88	$31.50	$162.10	$4,655.19
6	$101.38	$70.35	$31.03	$193.13	$4,584.84
7	$101.38	$70.82	$30.57	$223.70	$4,514.03
8	$101.38	$71.29	$30.09	$253.79	$4,442.74
9	$101.38	$71.76	$29.62	$283.41	$4,370.97
10	$101.38	$72.24	$29.14	$312.55	$4,298.73
11	$101.38	$72.72	$28.66	$341.21	$4,226.01
12	$101.38	$73.21	$28.17	$369.38	$4,152.80
13	$101.38	$73.70	$27.69	$397.07	$4,079.10
14	$101.38	$74.19	$27.19	$424.26	$4,004.91
15	$101.38	$74.68	$26.70	$450.96	$3,930.23
16	$101.38	$75.18	$26.20	$477.16	$3,855.05
17	$101.38	$75.68	$25.70	$502.86	$3,779.37
18	$101.38	$76.19	$25.20	$528.06	$3,703.18
19	$101.38	$76.69	$24.69	$552.75	$3,626.49
20	$101.38	$77.21	$24.18	$576.92	$3,549.28
21	$101.38	$77.72	$23.66	$600.59	$3,471.56
22	$101.38	$78.24	$23.14	$623.73	$3,393.33
23	$101.38	$78.76	$22.62	$646.35	$3,314.57
24	$101.38	$79.28	$22.10	$668.45	$3,235.28
25	$101.38	$79.81	$21.57	$690.02	$3,155.47
26	$101.38	$80.35	$21.04	$711.05	$3,075.12
27	$101.38	$80.88	$20.50	$731.55	$2,994.24

Figure 19: Amortization table

Example 3 **Reading from an Amortization Table**

Using the partial amortization table (Figure 6.23), answer the following questions.

Amortization Schedule Calculator					
Loan Amount		$10,000.00			
Interest Rate (annual)		4.75%			
Term (Years)		20			
Monthly Payment		$64.62			
Month	Payment	Principal	Interest	Total Interest	Balance
					$10,000.00
1	$64.62	$25.04	$39.58	$39.58	$9,974.96
2	$64.62	$25.14	$39.48	$79.07	$9,949.82
3	$64.62	$25.24	$39.38	$118.45	$9,924.59
4	$64.62	$25.34	$39.28	$157.74	$9,899.25
5	$64.62	$25.44	$39.18	$196.92	$9,873.81
6	$64.62	$25.54	$39.08	$236.01	$9,848.27
7	$64.62	$25.64	$38.98	$274.99	$9,822.63
8	$64.62	$25.74	$38.88	$313.87	$9,796.89
9	$64.62	$25.84	$38.78	$352.65	$9,771.05
10	$64.62	$25.95	$38.68	$391.33	$9,745.10
11	$64.62	$26.05	$38.57	$429.90	$9,719.05
12	$64.62	$26.15	$38.47	$468.37	$9,692.90
13	$64.62	$26.25	$38.37	$506.74	$9,666.65
14	$64.62	$26.36	$38.26	$545.00	$9,640.29
15	$64.62	$26.46	$38.16	$583.16	$9,613.83
16	$64.62	$26.57	$38.05	$621.22	$9,587.26
17	$64.62	$26.67	$37.95	$659.17	$9,560.59
18	$64.62	$26.78	$37.84	$697.01	$9,533.81
19	$64.62	$26.88	$37.74	$734.75	$9,506.92
20	$64.62	$26.99	$37.63	$772.38	$9,479.93
21	$64.62	$27.10	$37.52	$809.91	$9,452.84
22	$64.62	$27.20	$37.42	$847.32	$9,425.63
23	$64.62	$27.31	$37.31	$884.63	$9,398.32
24	$64.62	$27.42	$37.20	$921.83	$9,370.90
25	$64.62	$27.53	$37.09	$958.93	$9,343.37
26	$64.62	$27.64	$36.98	$995.91	$9,315.73
27	$64.62	$27.75	$36.87	$1,032.79	$9,287.98

Figure 20: Amortization table

1. What is the loan amount (principal), the interest rate, and the term of the loan?
2. How much is the monthly payment?
3. How much remaining balance is there after the payment in month 15?
4. How much was the interest in payment 10?
5. What is the total of the interest paid after payment 18?
6. What happens to the amount paid in interest each month?

Solution

1. Reading the values at the top of the table, we see the principal is $10,000, the interest rate is 4.75%, and has a term of 20 years.
2. The monthly payment is listed below the term of the loan, and is $64.62.
3. $9,613.83
4. $38.68

5. $697.01
6. The amount paid to interest decreases each month.

VIDEO
Reading an Amortization Table

Cost of Finance

There are often costs associated with a loan beyond the interest being paid. The **cost of finance** of a loan is the sum of all costs, fees, interest, and other charges paid over the life of the loan.

Example 4 **Cost of Financing a Personal Loan**

Irena signed for a loan of $15,000 at 6.33% for 5 years. When she took out the loan, Irena paid a $750 origination fee. Over the course of the loan, she pays $2,537.96 in interest. What was her cost to finance the loan?

Solution

The cost of finance is the sum is all interest and any fees paid for the loan. The fees paid were $750.00 and the interest was $2,537.96. Her cost of finance for this loan was $3,287.96.

Key Terms

- Fixed interest rate
- Variable interest rate
- Installment loan
- Loan amortization
- Revolving credit
- Amortization table
- Cost of finance

Key Concepts

- There are many reasons for a loan, but primarily it is taken out for a large expense when cash is not available.
- Each payment for an installment loan consists of an interest portion and a principal portion.
- There is a formula to calculate the payment necessary to pay off a loan in installments.
- Amortization schedules, or tables, show how each payment is applied to principal and interest. It also includes other details such as remaining balance and total interest paid.
- Loans often have other fees associated with them such as origination fees or application fees. The total of the interest paid and the fees is the cost of finance.

Video

- Credit Scores Explained
- Reading an Amortization Table

Formula

$$I = P \times \frac{r}{n}$$

$$pmt = \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n \times t}}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1}$$

6.9 Understanding Student Loans

Figure 21: College is often paid for through loans.

Learning Objectives

After completing this section, you should be able to:

1. Describe how to obtain a student loan.
2. Distinguish between federal and private student loans and state distinctions.
3. Understand the limits on student loans.
4. Summarize the standard prepayment plan.
5. Understand student loan consolidation.
6. Summarize and describe benefits or drawbacks of other repayment plans.
7. Summarize possible courses of action if a student loan defaults.

Obtaining a Student Loan

All college students are eligible to apply for a loan regardless of their financial situation or credit rating. Federal student loans do not require a co-signer or a credit check. Most students do not have a credit history when they begin college, and the federal government is aware of this. However, private loans will generally require a co-signer as well as a credit check. The co-signer will assume responsibility for paying off the loan if the student cannot make the payments.

The first step in applying for student loans is to fill out the **FAFSA** (Free Application for Student Aid). FAFSA determines financial need and what type of loan the student is qualified to obtain. For students who are still dependents on their parent's taxes, the parents also fill out the FAFSA, as their wealth and income impacts what the dependent student is eligible for. Students who cannot demonstrate financial need will also be helped by applying with FAFSA, as it will help guide them to the type of loan most appropriate. The FAFSA must be submitted each year.

CHECKPOINT

The FAFSA deadline is the spring of the student's next academic year. The deadline is often in March. Do not allow this deadline to pass.

As soon as an offer letter from the college is received, the student should start the application process. The college will determine the loan amount needed. Also, there are limits on the amount a student can borrow. There are both yearly limits and aggregate limits. See the table later in this section that outlines the loan limits per school year and in the aggregate.

If the student receives a direct subsidized loan, there is a limit on the eligibility period. The time limit on eligibility depends on the college program into which the student enrolls. The school publishes how long a program is expected to take. The eligibility period is 150% of that published time. For example, if a student is enrolled in a 4-year program, such as a bachelor's degree program, their eligibility period is 6 years, as $1.50(4) = 6$. Therefore, the student may receive direct subsidized loans for a period of 6 years.

Types and Features of Student Loans

Once a tuition statement is received, and all the non-loan awards are analyzed that are applicable to the costs of college (such as scholarships and grants), there still may be quite of bit of an expense to attend college. This difference between what college will cost (including tuition, room and board, books, computers) and the non-loan awards received is the **college funding gap.**

Example 1 **College Funding Gap**

Ishraq receives her award and tuition letter from the college she wants to attend. Her tuition, fees, books, and room and board all come to \$24,845 for the year. Her non-loan awards include an instant scholarship from the school for \$7,500, a scholarship she earned for enrolling in a STEM program for \$3,750, and a \$1,000 scholarship from her church. What is Ishraq's college funding gap?

Solution

Her awards total to $12,250. Her cost to attend is $24,845. Her college funding gap is then $\$ 24,845 - \$ 12,250 = \$ 12,595$. She will need to find $12,595 in funding.

There are several loan types, which basically break down into four broad categories: subsidized loans, unsubsidized loans, PLUS loans, and private loans. These loans are meant to fill the college funding gap.

Federal **subsidized loans** are backed by the U.S. Department of Education. These loans are intended for undergraduate students who can demonstrate financial need. Subsidized federal loans, including Stafford loans, defer payments until the student has graduated. During the deferment, the government pays the interest while the student is enrolled at least half-time. These loans are generally made directly to students. However, there are restrictions on how the money can be used. It can only be used for tuition, room and board, computers, books, fees, and college-related expenses. Interest rates are not based on the financial markets but determined by Congress. Federal loans are backed by the Department of Education.

Federal **unsubsidized loans**, including unsubsidized Stafford loans, are available for undergraduate and graduate students who cannot demonstrate financial need. If the student meets the program requirements, they are automatically approved. The student is not required to pay these loans during their time in college (enrolled at least half-time). However, the interest rate is generally higher and there is no deferment period, as with subsidized loans. Interest begins accruing as soon as the money is disbursed.

CHECKPOINT

The immediate accrual of interest means the balance of the loan grows as the student attends school. A loan that was for $10,000 can grow past $13,000 over five years of college. Some advisors tell students to pay the interest portion of the loan while it is deferred to prevent this growth of debt.

Parent Loans for Undergraduate Students (PLUS) are federal loans made directly to parents. They are available even if parents are not deemed financially needy. A credit check is performed and approval is not automatic. The limit to what parents can borrow from a PLUS loan each year is still the college funding gap, but the aggregate of the PLUS loans does not have a limit. This means the PLUS loan can cover whatever is left in the funding gap once all other aid and loans are applied. Payments do not begin until the student is out of school, but interest begins to accrue the moment funds are disbursed. Because the parents take out the loan, the parents are responsible for paying back the loan.

Private student loans are backed by a bank or credit institution and require a credit check, and interest rates are variable. As private loans are not subsidized by the government, no one pays the interest but the borrower. The student does not have to start repaying the loan until after graduation, but interest starts to accrue immediately. This loan has fewer repayment options, more fees and penalties, and the loan cannot be discharged through bankruptcy. Many students need a co-signer to acquire a private loan. Like PLUS loans, private student loans can cover whatever is left in the funding gap once all other aid and loans are applied

Student loans, in general, have a term of 10 years, that is, the loans are paid back over 10 years. This can vary, but 10 years is the standard.

WHO KNEW?

School-Channel Loans and Direct-to-Consumer Loans

Private loans can fall into one of two categories: school-channel loans and direct to consumer loans. **School-channel loans** are disbursed directly to the school. The school verifies the loan does not exceed the cost to attend school. **Direct–to-consumer loans** do not have the verification process. Those proceeds are sent directly to the borrower. They are processed more quickly, but often have higher interest rates.

VIDEO

Types of Student Loans

Limits on Student Loans

As mentioned earlier, there are limits to how much a student can borrow, per year and in total. The following table shows a general breakdown of the amounts the federal government and private lenders will lend. Amounts are based on level of need and whether the student is a dependent or an independent student. Independent students include those who are at least 24 years old, married, a professional, a graduate student, a veteran, a member of the armed forces, an emancipated minor, or an orphan. The amounts shown are as of this writing in 2022.

Year	Dependent Students Maximum Amounts	Independent Students Maximum Amounts
First-Year Undergraduate	\$5,500 but no more than \$3,500 may be in subsidized loans	\$9,500 but no more than \$3,500 may be in subsidized loans
Second-Year Undergraduate	\$6,500 but no more than \$4,500 may be subsidized loans	\$10,500 but no more than \$4,500 in subsidized loans
Third Year and Additional Years	\$7,500 but no more than \$5,500 may be in subsidized loans	\$12,500 but no more than \$5,500 may be in subsidized loans
Graduate and Professional	Not applicable	\$20,500 in unsubsidized loans
Limits	\$31,000 but no more than \$23,000 in subsidized loans	\$57,500 for undergraduates but no more than \$23,000 in subsidized. \$138,500 for graduate or professional but no more than \$65,500 may be subsidized loans.

Check out this Edvisors page on the limits of student borrowing to learn more!

Example 2 **Loan for Year 5 of College**

Efraim is a dependent undergraduate student enrolled in a biology program. He's about to attend for the fifth year. In year 1 he took out \$5,000 in federal subsidized and unsubsidized loans, in year 2 he took out \$6,400 in federal subsidized and unsubsidized loans, in years 3 and 4, he took out the maximum federal subsidized and unsubsidized loans amounts. He needs federal subsidized and unsubsidized loans for his fifth year of school. How much can he obtain in federal subsidized and unsubsidized student loans?

Solution

The sum of his previous loans is $\$\,5,000+\$\,6,400+\$\,7,500+\$\,7,500=\$\,26,400$. The limit for federal subsidized and unsubsidized loans is \$31,000, so in year 5 he can get student loans in the amount of $\$\,31,000-\$\,26,400=\$\,4,600$.

Putting this all together, we have a way to determine the student loans needed for a student to attend college.

- First, determine the funding gap. If the student or family can cover the gap, then no loans are necessary.

- Second, determine how much in federal subsidized and unsubsidized student loans can be taken out. If the total federal loans available is more than the funding gap, no other loans are needed.
- Third, if the federal subsidized and unsubsidized loans do not cover the gap, PLUS and private student loans can be taken out to cover the remainder of the gap.

  At each step, if the student and family can cover some or all of the gap, they can do so without taking out a loan.

Example 3 **College Funding Gap and PLUS and Private Student Loans**

Olivia receives her award and tuition letter from the college she wants to attend. Her tuition, fees, books, and room and board all come to $44,845 for her second year. Her non-loan awards include an instant scholarship from the school for $13,500, a scholarship she earned for enrolling in an engineering program for $5,750, and a $2,000 scholarship from her parent's workplace. For her first year, what is Olivia's college funding gap? How much can Olivia borrow in federal subsidized and unsubsidized student loans? Once Olivia takes out her maximum subsidized and unsubsidized federal student loans, how much will have to be paid for using PLUS and private student loans?

Solution

Her awards total to $21,250. Her cost to attend is $44,845. Her college funding gap is then $\$\,44,845 - \$\,21,250 = \$\,23,595$. The maximum in federal student loans that Olivia can borrow is $6,500 in year 2. The remaining funding gap is $\$\,23,595 - \$\,6,500 = \$\,17,095$. Private student loans, PLUS loans, or other sources must be used to cover this gap.

Student Loan Interest Rates

Student loans are first and foremost loans. Students will pay them back and will pay interest. In the fall of 2022, the federal student loan interest rate was 4.99%. Private student loans rates ranged between 3.22% and 13.95%. Finding the lowest interest rate you can helps with the payments, and especially helps if the loan is not federally subsidized. Remember, if the loan is not federally subsidized, the student is on the hook for the interest that is accumulating with the loan.

Interest Accrual

The interest on student loans begins as soon as the loan is disbursed (paid to the borrower). When the loan is federally subsidized, the government pays that interest for the student. This means the loan for a subsidized loan of $3,000 is still a loan for $3,000 when the student graduates. However, if the loan is not federally subsidized, the student is responsible for the interest that accrues on the loan. The $3,000 loan from year 1 of college is now a loan for more due to that added interest. The interest on that loan grew while the student was in college. The formula for growth of the loan's balance is the same as compound interest formula from Compound Interest, $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

Example 4

Denise takes out unsubsidized student loan, in August, in her first year of college for $2,000. She manages an interest rate of 8%. She graduates after her fifth year of college, in May. She does not pay the interest on the loan during her time in college. What is the balance of her first year loan in May of her graduation year?

Solution

The principal of the loan is $2,000. Her interest rate is 8%. Since student loans are typically paid monthly, there are 12 periods per year. Since the time she has had the loan is not in years, we will use the number of months for the value of *nt* in the formula. She has had the loan for 4 years and 9 months, meaning 57 period have passed. Substituting those values into the formula and calculating, we find her balance in May of her graduating year is $A = P\left(1+\frac{r}{n}\right)^{nt} = \$\,2,000\left(1+\frac{0.08}{12}\right)^{57} = \$\,2,000\left(1.0\overline{6}\right)^{57} = \$\,2,920.89$.

The Standard Repayment Plan

There are various repayment plans available. The one most likely to apply to a student loan is the **standard repayment plan,** which is available to everyone. Borrowers pay a fixed amount monthly so the loan is paid in full within 10 years. Consolidated loans, discussed later in this section, also qualify for the standard repayment plan, and may allow the payoff period to range from 10 to 30 years. Direct subsidized and unsubsidized loans, PLUS loans, and federal Stafford loans are eligible.

Since these are loans, they are paid back with interest. As with most installment loans, their payments are due monthly. The formula for paying back these loans is the same as the formula used for paying loans in The Basics of Loans:

$$pmt = \frac{P \times \left(\frac{r}{n}\right) \times \left(1+\frac{r}{n}\right)^{n\times t}}{\left(1+\frac{r}{n}\right)^{n\times t} - 1}$$

Using that formula, we can calculate how much the payment is for a student loan. Remember that all loan payments are rounded up to the next penny.

Example 5 **Standard Repayment Plan**

Find the payment for the following student loans using the standard repayment plan:

1. Loan is $3,500, interest is 4.99%
2. Loan is for $6,200, interest is 6.75%

Solution

1. The principal is $P = \$3{,}500$ and the rate is $r = 0.0499$. Since this is the standard repayment plan, there are $n = 12$ payments per year for 10 years. Substituting those values into $pmt = \frac{P\times\left(\frac{r}{n}\right)\times\left(1+\frac{r}{n}\right)^{n\times t}}{\left(1+\frac{r}{n}\right)^{n\times t}-1}$ and calculating gives a monthly payment of

$$
\begin{aligned}
pmt &= \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n \times t}}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1} \\
&= \frac{\$\ 3{,}500 \times \left(\frac{0.0499}{12}\right) \times \left(1 + \frac{0.0499}{12}\right)^{12 \times 10}}{\left(1 + \frac{0.0499}{12}\right)^{12 \times 10} - 1} \\
&= \frac{\$\ 3{,}500 \times \left(0.0041583\overline{3}\right) \times \left(1.0041583\overline{3}\right)^{120}}{\left(1.0041583\overline{3}\right)^{120} - 1} \\
&= \frac{\$\ 23.9469911276}{0.645370131872} \\
&= \$\ 37.11
\end{aligned}
$$

2. The principal is $P = \$6{,}200$ and the rate is $r = 0.0675$. Since this is the standard repayment plan, there are $n = 12$ payments per year for 10 years. Substituting those values into $pmt = \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n \times t}}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1}$ and calculating gives a monthly payment of

$$
\begin{aligned}
pmt &= \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n \times t}}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1} \\
&= \frac{\$\ 6{,}200 \times \left(\frac{0.0675}{12}\right) \times \left(1 + \frac{0.0675}{12}\right)^{12 \times 10}}{\left(1 + \frac{0.0675}{12}\right)^{12 \times 10} - 1} \\
&= \frac{\$\ 6{,}200 \times (0.005625) \times (1.005625)^{120}}{(1.005625)^{120} - 1} \\
&= \frac{68.3662233426}{0.960321816275} \\
&= \$\ 71.20
\end{aligned}
$$

Example 6 **Standard Repayment Plan for an Unsubsidized Loan**

Erson has a balance of $8,132.55 when he starts paying off the 8.6% unsubsidized student loan he took out in his third year. How much are his payments if the term for his loan is the standard 10 years?

Solution

Using the payment formula, $pmt = \frac{P\times\left(\frac{r}{n}\right)\times\left(1+\frac{r}{n}\right)^{n\times t}}{\left(1+\frac{r}{n}\right)^{n\times t}-1}$, with $P = \$8{,}132.55$, $r = 0.086$, $t = 10$ and $n = 12$, we calculate that his monthly payment will be

$$\begin{aligned} pmt &= \frac{P\times\left(\frac{r}{n}\right)\times\left(1+\frac{r}{n}\right)^{n\times t}}{\left(1+\frac{r}{n}\right)^{n\times t}-1} \\ &= \frac{\$\,8{,}132.55\times\left(\frac{0.086}{12}\right)\times\left(1+\frac{0.086}{12}\right)^{12\times 10}}{\left(1+\frac{0.086}{12}\right)^{12\times 10}-1} \\ &= \frac{\$\,8{,}132.55\times\left(0.0071\overline{6}\right)\times\left(1.0071\overline{6}\right)^{120}}{\left(1.0071\overline{6}\right)^{120}-1} \\ &= \frac{137.310962384}{1.35592393158} \\ &= \$\,101.27 \end{aligned}$$

Student Loan Consolidation

When a student graduates, they may have multiple different student loans. Keeping track of them and paying them off separately can be a burden. Instead, these loans can be consolidated into a single loan. If they are federal loans the combination is called **federal consolidation.** Combining private loans is often referred to as **refinancing.** Refinancing, or **private consolidation,** can be used to combine both private and federal student loans. Be aware that consolidated federal loans may still be subject to the rules and protections that govern subsidized loans. Refinancing loans, private or federal, are no longer subject to those rules and guidelines. Check out this Experian article about consolidation and refinancing for more deatil.

In consolidation of federal direct student loans, the interest rate is the weighted average of the interest rates on the subsidized loans. This means the interest rate remains the same. However, if the term is extended, then the student will pay back more over time than if they did not extend the loan term.

In refinancing, it is possible to obtain a lower interest rate on the student loans, which may lower how much is paid per month and lower the total paid back over time. These monthly payments are calculated using the same formula as for any other loan payment, $pmt = \frac{P\times\left(\frac{r}{n}\right)\times\left(1+\frac{r}{n}\right)^{n\times t}}{\left(1+\frac{r}{n}\right)^{n\times t}-1}$. The term of the refinanced loan may also be changed, which would also impact the payment per month.

In either case, consolidating or refinancing, the monthly financial burden on the student can decrease. However, if the term is extended, the total amount repaid may increase.

Example 7 **Federal Loan Consolidation and Interest Rates**

Ernest has four federal student loans that he wants to consolidate. He combines them into one loan. What is the maximum Ernest can reduce the interest rate by?

Solution

Consolidating subsidized loans has no impact on the interest rate of the loans, so the maximum that the interest rate can be reduced is 0%.

Example 8 **Payments for Consolidated Student Loans**

Brianna consolidates her student loans, some federal and some private, into a single refinanced student loan with a principal of $27,800. The interest rate that Brianna received was 8.375%. If Brianna's new term is 15 years, how much are her payments per month?

Solution

The principal is $P = \$27{,}800$ and the rate is $r = 0.08375$. Since the payments are monthly, $n = 12$. The loan term is for 15 years, so $t = 15$. years. Substituting those values into $pmt = \frac{P \times \left(\frac{r}{n}\right) \times \left(1+\frac{r}{n}\right)^{n \times t}}{\left(1+\frac{r}{n}\right)^{n \times t} - 1}$ and calculating gives a monthly payment of

$$\begin{aligned} pmt &= \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n \times t}}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1} \\ &= \frac{\$\ 27{,}800 \times \left(\frac{0.08375}{12}\right) \times \left(1 + \frac{0.08375}{12}\right)^{12 \times 15}}{\left(1 + \frac{0.08375}{12}\right)^{12 \times 15} - 1} \\ &= \frac{\$\ 6{,}200 \times \left(0.0069791\overline{6}\right) \times \left(1.0069791\overline{6}\right)^{180}}{\left(1.0069791\overline{6}\right)^{180} - 1} \\ &= \frac{\$\ 678.477992464}{2.49693370968} \\ &= \$\ 271.73 \end{aligned}$$

Other Repayment Plans

There are various other repayment plans available to students. Plans other than the standard repayment plan typically require the student to meet certain criteria. The following plans are independent of student income, but may make early payments easier.

- **Graduated repayment plans** are plans where the amount of payments gradually increases so that the loan is paid off in 10 years, or within 10 to 30 years for consolidated loans. Payments start off small and increase approximately every 2 years. Almost all loan types are eligible, including direct subsidized and unsubsidized loans, Stafford loans, PLUS loans, and consolidated loans.

- **Extended repayment plans** are available to the direct loan borrower if the outstanding direct loans are over $30,000. The payments, fixed or graduated, are designed so that the loans are satisfied within 25 years. Eligible loans include both direct subsidized or unsubsidized loans, Stafford loans, PLUS loans, and consolidated loans.

  If student earnings are such that the standard, graduated, or extended repayment plans are unaffordable, then one can make payments that are based on their **discretionary income.** Discretionary income is federally defined to be the difference between (adjusted) gross income and 150% of the poverty guideline for location and family size. This discretionary income then depends on where one lives (contiguous United States or Hawaii or Alaska) and how many dependents one has. If married, a spouse's income will be included in the adjusted gross income. Understanding discretionary income is necessary to understand how income driven payments plans work.

Example 9 **Discretionary Income**

1. The poverty guideline for a single person living in Arkansas, is $12,000. If Harriet is a single person in Arkansas with a (adjusted) gross income of $23,500, what is her discretionary income?
2. For California, the poverty guideline for a person with four people in the household is $27,750. If such a person has a (adjusted) gross income of $48,600, what is their discretionary income?

Solution

1. The poverty guideline for a single person in Arkansas is $12,000. 150% of that guideline value is $1.5 \times \$\,12,000 = \$\,18,000$. The gross income that Harriet makes over that $18,000 is her discretionary income. That gross income is $23,500, so her discretionary income is $\$\,23,500 - \$\,18,000 = \$\,5,500$.
2. The poverty guideline for a household of four in California is $27,750. 150% of that guideline value is $1.5 \times \$\,27,750 = \$\,41,625$. The gross income that Harriet makes over that $41,625 is her discretionary income. That gross income is $48,600, so her discretionary income is $\$\,48,600 - \$\,41,625 = \$\,6,975$.

The following plans all depend on discretionary income.

- **Pay As You Earn (PAYE) repayment plans** have monthly payments that are 10% of discretionary income based on a student's updated income and family size. If a borrower files a joint tax return, their spouse's income and debt may also be considered. Eligible loans include direct subsidized and unsubsidized loans, PLUS loans made to students, and some consolidated loans. These loans are forgiven (student does not pay the remaining balance) after 20 years of monthly payments if they were direct federal student loans.

- **Revised Pay As You Earn (REPAYE) repayment plans** have payment amounts that are based on income and family size and calculated as 10% of discretionary income. Eligible loans include direct subsidized and unsubsidized loans, direct PLUS loans, and some consolidated loans. These loans are forgiven, that is, the student does not pay more, after 20 or 25 years, provided they were direct federal student loans.

- **Income-Based Repayment (IBR) plans** sound like a few of the others and there are similarities in all of them. The payments are either 10% or 15% of discretionary income, but this plan is meant for those with a relatively high debt. Every year, income and family size must be updated, and payments are calculated based on those figures. Eligible loans include direct subsidized and unsubsidized loans, Stafford loans, and PLUS loans made to students, but not PLUS loans made to parents.
- **Income-Contingent Repayment (ICR) plans** have payments that are either 20% of discretionary income or whatever would be paid if a student were on a fixed payment plan for more than 12 years, whichever is less. Eligible loans include direct subsidized and unsubsidized, PLUS loans to students, and consolidated loans.

There are many similarities among these repayment plans, and it is easy to misunderstand the nuances of each. Therefore, be careful entering into any type of repayment contract until you fully understand all the details and repercussions of the plan you choose. For more detail, see this nerdwallet article "Income-Driven Repayment: Is It Right for You?" to learn more!

Example 10 **Payments for a REPAYE Program**

Warren qualifies for a REPAYE payment plan. His gross income is $32,700. He is single and live in Montana, so the federal poverty guideline for Warren is $12,000.

1. What is Warren's discretionary income?
2. Under the REPAYE plan, he pays 10% of his discretionary income, but monthly. How much are Warren's REPAYE payments?

Solution

1. The poverty guideline for Warren is $12,000. 150% of that guideline value is $1.5 * \$\,12,000 = \$\,18,000$. The gross income that Warren makes over that $18,000 is his discretionary income. That gross income is $32,700, so his discretionary income is $\$\,32,700 - \$\,18,000 = \$\,14,700$.
2. 10% of Warren's discretionary income is $0.1 \times \$\,14,700 = \$\,1,470$. He pays monthly, so his monthly payments are $1,470 divided by 12, or $122.50 per month.

Using income to determine payments initially seems excellent. However, if there is no forgiveness at the end of the loan, then the income driven payment plans can cause problems. For one, your payment may not be sufficient to cover the interest rate of your loans. In that case, your loan balance actually *increases* as you make your payments. Eventually, you are paying for not only your original loan balance, but interest that has been growing and compounding over time. Also, if the loan term is extended, you may pay more, perhaps a lot more, money over time. You may find yourself in the position of paying these loans for decades.

With those possible drawbacks, great care must be taken to avoid large problems down the line.

> **VIDEO**
> Repayment Plans

Student Loan Default and Consequences

The first day a payment is late, the account becomes **delinquent**. After 90 days, this delinquency is reported to the credit bureaus, and goes into **default**. This is serious, as now a credit score is affected, meaning that it will be harder to buy a car, a home, get a credit card, or a cell phone. Even renting an apartment may be a task not easily overcome. The default rate for students who do not complete their degree is three times higher than for students who do.

Further, defaulting on a student loan may mean that the borrower loses eligibility for repayment plans, as the balance and any unpaid interest may become due immediately, and any tax refunds may be withheld and applied to the loan, and wages may be garnished. One should immediately contact your loan servicer and try to make other arrangements for repayment if this situation becomes apparent, as different repayment plans are available, if actions are taken quickly.

There are several options that may be open to avoid defaulting. One is called **rehabilitation,** or is the process in which a borrower may bring a student loan out of default by adhering to specified repayment requirements, and the other is consolidation. Certain criteria must be met to enter these programs.

Both of these options are detailed, including the criteria required for eligibility, on the studentaid.gov loan management page.

Professionals advise hiring an attorney if one of these paths is chosen.

Key Terms

- FAFSA
- College funding gap
- Subsidized loan
- Unsubsidized loan
- Parent loan for undergraduate students
- Private student loan
- School-channel loan
- Direct-to-consumer loan
- Standard repayment plan
- Federal consolidation
- Refinancing
- Private consolidation
- Graduated repayment plan
- Extended repayment plan

- Discretionary income
- Pay as you earn (PAYE) repayment plan
- Revised pay as you earn (REPAYE) repayment plan
- Income-based (IBR) repayment plan
- Income-contingent (ICR) repayment plan
- Delinquent
- Default
- Rehabilitation

Key Concepts

- The FAFSA must be filled out each year that a student wishes to borrow for.
- A student's funding gap determines how much they need in loans to pay for college.
- Federal subsidized student loans defer payments until after graduation and interest does not accrue on these loans.
- Unsubsidized student loans defer payment until after graduation but interest begins accruing as soon as the loan finds are disbursed.
- There are both yearly and aggregate limits for student loans to prevent over-borrowing, among other reasons.
- Federal direct loans have a low interest rate set by the government, but other student loans have varying rates of interest set by the banks.
- The standard repayment plan lasts 10 years and is made up of monthly payments.
- Consolidating or refinancing student loans merges many student loans into one loan.
- If only federal loans are consolidated, the interest rate is the same as the individual loans, currently set at 4.99%.
- If other loans are refinanced together, the interest rate may be lower with the new loan.
- Other repayment plans are available. Such a plan may have payment that start small and grow as the loan is paid off, or it may have a longer term, or may be based on the discretionary income of the student.
- Being delinquent on a student loan is a precursor to being in default. Making payments in a timely fashion allows the student to avoid this situation.

Video

- Types of Student Loans
- Repayment Plans

Formula

$$\text{funding gap} = \text{total cost} - \text{all aid}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$pmt = \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n \times t}}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1}$$

$$\text{discretionary income} = \text{gross income} - 1.5 \times \text{poverty guideline}$$

6.10 Credit Cards

Figure 22: Credit cards are both a convenience and a danger. (credit: “Credit Cards” by Sean MacEntee/Flickr, CC BY 2.0)

Learning Objectives

After completing this section, you should be able to:

1. Apply for a credit card armed with basic knowledge.
2. Distinguish between three basic types of credit cards.
3. Compare and contrast the benefits and drawbacks of credit cards.
4. Read and understand the basic parts of a credit card statement.
5. Compute interest, balance due, and minimum payment due for a credit card.

It can be difficult to get along these days without at least one credit card. Most hotels and rental car agencies require that a credit card is used. There are even a number of retailers and restaurants that no longer accept cash. They make online purchasing easier. And nothing contributes more to a good credit rating than a solid history of making credit card payments on time.

Being granted a credits card is a privilege. Used unwisely that privilege can become a curse and the privilege may be withdrawn. In this section, we will talk about the different types of credit cards and their advantages and disadvantages. The more knowledge a cardholder has about the credit card industry, the better able credit accounts can be managed, and that knowledge may cause major adjustments to a cardholder's lifestyle.

All credit cards are not equal, but they all represent consumers borrowing money, usually from a bank, to pay for needs and "wants." As such, they are a type of loan, and your repayment may include interest. (You might want to review Section 6.8, which discusses loans and repayment plans.)

There are many institutions and credit cards to choose from. Use caution as you shop around for a credit card that suits you. Your top concern is likely the interest rates on purchases and cash advances. But be careful to also read the small print regarding charges for late payments, and other fees such as an **annual fee**, where the credit card charges you (the cardholder) a fee each year for the privilege of using the cards. Many cards charge no such fee, but there are many that charge modest to heavy fees. Make sure to understand rules for **reward programs**, where the credit card issuer grants benefits based on one's spending. Finally, once one applies for and is granted a credit card, pay attention to the **credit limit** the bank offers. Once a company is owed that much money, use of the card for purchases should be curtailed until some of the debt is paid off.

> **CHECKPOINT**
> *The interest rate will not matter if the balance is paid every month. When the balance is paid every month, there is NO INTEREST charged.*

Types of Credit Cards

There are basically three types of credit cards: bank-issued credit cards, store-issued credit cards, and travel/entertainment credit cards. We will look at all three and explain the good and the bad qualities of each.

Bank-Issued Credit Cards

Perhaps the most widely used credit card type is the **bank-issued credit card**, like Visa or MasterCard (and even American Express and Discover cards). These types of cards are an example of revolving credit, meaning that additional credit is extended before the previous balance is paid—but only up to the assigned credit limit. Bank-issued cards are considered the most convenient, as they can be used to purchase anything, including apparel, furniture, groceries, fuel for automobiles, meals, hotel bills, and so on, just as if paying with cash. The interest rates on bank-issued credit cards are usually lower than those for other credit cards we'll discuss, and the credit limits are generally higher. Currently, bank-issued cards have an average 20.09% APR.

Store-Issued Credit Cards

Store-issued credit cards are issued by retailers. One can hardly walk into a store these days without being offered a discount on purchases if one applies for the store credit card. These cards can only be used in that store or family of stores that issues the card. However, if a store credit card is associated with Visa, MasterCard, or American Express, then the card might be used the same way that the bank-issued cards are used. This is called cobranding. The logo of the bank-issued card will be present on the store card. Many stores offer both types. Like other credit cards, they may come with an annual fee.

Store credit cards usually charge higher interest rates than bank-issued cards. Currently, store credit cards have an APR (annual percentage rate) of 24.15%. Any rewards offered by store credit cards are usually limited to purchases made in their own store, and it typically takes longer to accumulate enough rewards or points to redeem them, whereas cobranded credit cards offer opportunities to earn rewards on all purchases, regardless of whether purchases are made in the issuing store or not.

Store credit cards usually offer lower credit limits, at least in the beginning. After being proved to be a responsible credit card owner, credit limits can be raised. Nevertheless, store credit cards are a good choice for those new to the credit card industry. If on-time payments are consistently made, it is an excellent way to get started building a credit history.

Travel/Entertainment Cards, or Charge Cards

This is the third type of credit card. The **travel and entertainment cards,** also known as**charge cards,** first and foremost offer very high limits or unlimited credit, but they must be paid in full every month. They generally charge high annual fees and impose expensive penalties should a payment be late. On the other hand, they typically have longer grace periods and offer many and various kinds of rewards.

Check out this nerdwallet article about the differences between a charge card and a credit card.

CHECKPOINT

An interest rate will greatly depend on credit score. Responsible use of credit cards will increase a credit score. See the WHO KNEW? from The Basics of Loans.

VIDEO

Choosing a Credit Card

WHO KNEW?

Top Travel and Entertainment Cards

At one time, Diner's Club was the premier entertainment card. To be accepted into the Diner's Club and be rewarded with a charge card meant one was special. Today, an American Express card is held with the same reverence as Diner's Club was in the past. It was a privilege to own one of these cards. There is a lot of competition going on to supplant Diner's Club, as shown by the Chase Sapphire Preferred Card's travel rewards and benefits.

Another thing worth mentioning here and something that appears to be an unusual offering is part of the American Express Gold and higher-level cards. Cardholders can actually open a savings account, buy a CD, or apply for a personal or business loan from American Express. The company boasts a higher interest rate than what is paid on traditional savings accounts and CDs.

Example 1 **Comparing Credit Cards**

1. Which type of credit card is paid off every month so has no interest to be paid, but comes with high fees?
2. Which type of credit cards are the most widely accepted?
3. Which type of credit cards are the most limited?

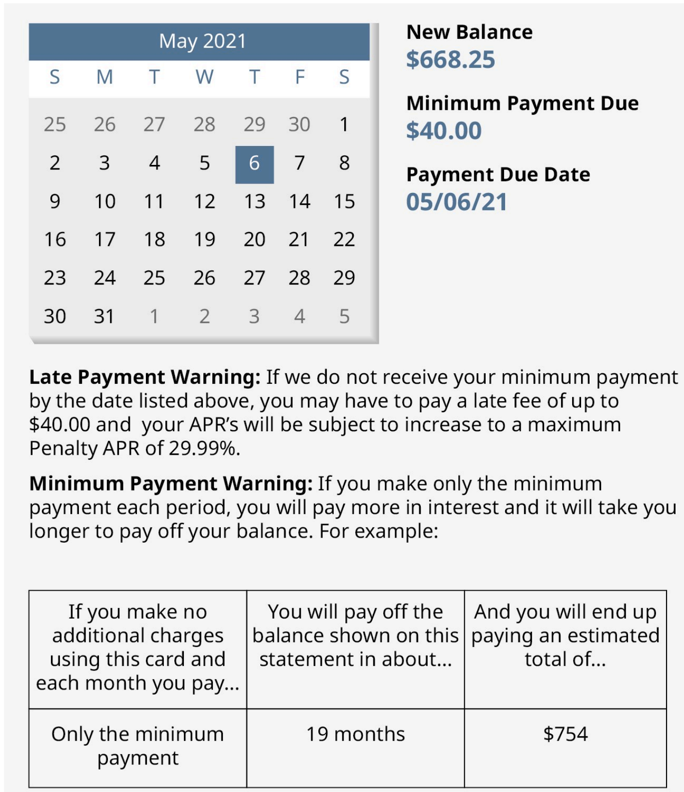
Solution

1. Charge cards are to be paid off completely each month.
2. Bank-issued credit cards are the most flexible to use, because they are not limited to which retailers or service providers accept them.
3. Store issues credit cards are the most limited, since they only work in that family of stores.

Credit Card Statements

Cardholders usually receive monthly statements and have 21 days to pay the minimum amount due. The statements itemize and summarize activity on the credit card for that statement's **billing period**. The billing period for a credit card is generally a month long, but typically does not start and end on the first and last days of the month. The statement will include the current balance, interest rate, the minimum payment due, and the due date. Be aware, different companies produce statements that are laid out differently. The information will be clearly labeled though.

The due date is a top concern. Missing a due date is one of the worst things a cardholder can do financially, and this is by far the biggest downfall of owning a credit card. Not only is the cardholder subject to late fees, but when a payment is late more than once there is a high probability that the cardholder will be negatively reported to the credit bureaus, which can quickly erode a credit score. shows an excerpt from an actual statement from a Chase Bank Visa card, based on the current $668.25 balance.



May 2021						
S	M	T	W	T	F	S
25	26	27	28	29	30	1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31	1	2	3	4	5

New Balance
$668.25

Minimum Payment Due
$40.00

Payment Due Date
05/06/21

Late Payment Warning: If we do not receive your minimum payment by the date listed above, you may have to pay a late fee of up to $40.00 and your APR's will be subject to increase to a maximum Penalty APR of 29.99%.

Minimum Payment Warning: If you make only the minimum payment each period, you will pay more in interest and it will take you longer to pay off your balance. For example:

If you make no additional charges using this card and each month you pay...	You will pay off the balance shown on this statement in about...	And you will end up paying an estimated total of...
Only the minimum payment	19 months	$754

Figure 23: Credit card statement

Specifically pay attention to the late payment penalty and minimum payment warning statements. stating that if no other purchases are made and you continue making only the minimum payment, it will take 19 months to pay off the balance and you will pay $754.00. You can't say you were not warned.

It is critical that you examine your statement every month because it is always a possibility that your account may have been compromised. If you should notice fraudulent charges on your statement, notifying the credit card company is often enough to have those charges researched by the company and removed. The card with the fraudulent charges will be canceled and a new card with a new account number will be sent to you.

Example 2 **Reading a Credit Card Statement**

On the credit card statement , identify

1. The balance due
2. The minimum required payment
3. The length of time it takes to pay off the balance by paying the minimum payments and without charging more to the card

4. The interest rate for purchases

YourBank

Your Credit Card Account Statement

Statement Period:
From April 01, 2015
To April 30, 2015

Account Number:
9999 99XX XXXX 1234

Primary Cardholder:
Yourname Yoursurname

• PAYMENT INFORMATION

New Balance:	$ 3663.23
Your Minimum Payment:	$ 36.63
Your Minimum Payment Due Date:	May 24, 2015

Estimated time to pay

The estimated time to pay your New Balance in full if you pay only the Minimum Payment each month is 2 year(s) and 4 month(s).

• SUMMARY OF YOUR ACCOUNT

Previous Statement Balance:	$ 2654.48
Payments:	$-2654.48
New Purchases:	$ 1957.24
Balance Transfers and Access Cheques:	$ 1200.00
Cash Advances	$ 500.00
Interest:	$ 0.00
Fees:	$ 5.99
Subtotal	$ 3663.23
Your New Balance:	$ 3663.23
Credit Limit:	$ 9000.00
Cash Advance Limit:	$ 500.00
Credit Available:	$ 5336.77
Cash Advance Limit:	$ 0.00
Statement Closing Date:	April 30, 2015
Days in Statement Period:	30
Annual Interest Rate for Purchases:	19.99%
Annual Interest Rate for Balance Transf. and Access Cheques:	2.50%
Annual Interest Rate for Cash Advances:	19.99%

Ways to pay:
Online Banking
Telephone Banking
ATM
Pie-Authorized Payment
By Mail To:
YourBank
P.O. Box 1234 Section Z
BankCity, NY, 98456

Contact Information:
www.websitename.com
Customer Service/Lost or Stolen
1-888-123-4567
TTY/TDD
1-888-123-4567

YourBank Payment Slip

PAYEE FIRST NAME LAST NAME
PAYEE ADDRESS:
99 STREET NAME
CITY NAME, ZIP CODE
STATE, COUNTRY

Account Number:	9999 99XX XXXX 123
Your New Balance:	$ 3663.23
Your Minimum Payment:	$ 36.63
Your Minimum Payment Due Date:	May 24, 2015
Amount you're paying:	$ 3663.23

999 999 999 99999 999 9999 99XX XXXX 1234 XX

Figure 24: Credit card account statement

Solution

1. The balance due is under the payment information heading and is $3,663.23.
2. The minimum payment due is also under the payment information heading, and is $36.63.
3. The time to pay off the balance using only minimum payments is below the payment information, and says it takes 2 years and 4 months to pay off the balance.
4. The interest rate for purchases is toward the bottom of the statement. It is 19.99%.

VIDEO

Reading Credit Card Statements

Compute Interest, Balance Due, and Minimum Payment Due for a Credit Card

Computing all of these values depends on understanding and computing the average daily balance on a credit card. Once that is known, the interest, balance due, and minimum payment can be found.

Above all else, if you pay off the entire balance each month, interest is not charged.

Average Daily Balance

Most credit card companies compute interest using the **average daily balance** method.

To find the average daily balance on your credit card, determine the balance on the card each day of the billing period (often that month), and take the average. One process to find that average daily balance follows these steps:

1. Start with a list of transactions with their dates and amounts.
2. For each day that had transactions, find the total of the transactions for the day. Expenditures are treated as positive values, payments are treated as negative values.
3. Create a table containing each day with a different balance. The balance is the previous balance plus the day's total transactions.
4. Add a column for the number of days those balances until the balance changed.
5. Add a column that contains the balances multiplied by the number of days until the balance changed.
6. Find the sum of that last column.
7. Divide the sum by the number of days in the billing period (often the number of days in the month). This is the average daily balance.

Example 3 **Computing Average Daily Balance**

The billing cycle goes from May 1 to May 31. The balance at the start of the billing cycle is $450.21. The list of transactions on the card is below.

Date	Activity	Amount
1-May	Billing Date Balance	$450.21
10-May	Payment	$120.00
15-May	Groceries	$83.43
26-May	Auto Parts	$45.12
26-May	Restaurant	$85.34
30-May	Shoes	$98.23

Find the average daily balance for the credit card during the month of May.

Solution

To find the average daily balance, we use the following steps.

1. Start with a list of transactions with their dates and amounts.

This list is provided.

1. For each day that had transactions, find the total of the transactions for the day.
   The only day with more than one transaction was May 26. The sum of those transactions is $130.46. Treating the payment as a negative value, the daily transaction amounts are

Date	Amount
1-May	$450.21
10-May	-$120.00
15-May	$83.43
26-May	$130.46
30-May	$98.23

2. Create a table containing each day with a different balance.
   The new table with dates that had different balances is below.

Date	Balance
1-May	$450.21
10-May	$330.21
15-May	$413.64
26-May	$544.10
30-May	$642.33

3. Now, add a column for the number of days those balances until the balance changed.
   The days until the balance changes is found by finding the difference in the dates. For instance, from May 15 to May 26 was 11. Adding that column to the table we have

Date	Balance	Days Until Balance Changes
1-May	$450.21	9
10-May	$330.21	5
15-May	$413.64	11
26-May	$544.10	4
30-May	$642.33	2

   The last entry was 2 since there are 31 days in May.

4. Add a column that contains the balances multiplied by the number of days until the balance changed. We create the column and multiply the values.

Date	Balance	Days Until Balance Changes	Balance Times Days
1-May	\$450.21	9	\$4,051.89
10-May	\$330.21	5	\$1,651.05
15-May	\$413.64	11	\$4,550.04
26-May	\$544.10	4	\$2,176.40
30-May	\$642.33	2	\$1,284.66

5. Find the sum of that last column. Adding that last column we have a sum of \$13,714.04.
6. There are 31 days in May, so divide the sum by 31, which gives an average of \$442.39, which is the average daily balance.

Calculating the Interest for a Credit Card

The interest charged for a credit card is based on the daily interest rate of the card, the number of days in the billing cycle, and the average daily balance on the card.

FORMULA

The interest charge, I, for a credit card during a billing cycle is $I = \frac{\text{ADB} \times r \times d}{365}$, where ADB is the average daily balance, r is the annual percentage rate, and d is the number of days in the billing cycle. As before, interest is rounded up to the next penny.

Example 4 **Calculating Interest for a Credit Card Billing Cycle**

Compute the interest charged for the credit card based on the given average daily balance (ABD), annual interest rate, and number of days in the billing cycle.

1. ADB = \$2,765.00, annual interest rate 13.99%, billing cycle of 30 days
2. ADB = \$789.30, annual interest rate 17.99%, billing cycle of 31 days
3. ADB = \$1,037.85, annual interest rate 11.99%, billing cycle of 28 days

Solution

1. Substituting \$2,765.00 for ADB, 0.1399 for r and 30 for d and calculating, we find the interest charge to be $I = \frac{\text{ADB} \times r \times d}{365} = \frac{\$\,2{,}765.00 \times 0.1399 \times 30}{365} = \$\,31.80$.
2. Substituting \$789.30 for ADB, 0.1799 for r and 31 for d and calculating, we find the interest charge to be $I = \frac{\text{ADB} \times r \times d}{365} = \frac{\$\,789.30 \times 0.1799 \times 31}{365} = \$\,12.06$.

3. Substituting $1,037.85 for ADB, 0.1199 for r and 28 for d and calculating, we find the interest charge to be $I = \frac{\text{ADB} \times r \times d}{365} = \frac{\$\,1{,}037.85 \times 0.1199 \times 28}{365} = \$\,9.55$.

WHO KNEW?

Credit Cards Charge Stores Fees

The interest you pay is not the only way a credit card company generates revenue. It also charges fees to the retailers, online stores, and service providers that allow the consumer, you, to use your credit card to pay them. These are called processing fees. Currently they typically range from 2.87% to 4.35% of each transaction. That means if you use your credit card at a store and spend $100.00, the store will have to pay the credit card company somewhere between $2.87 and $4.35.

One type of processing fee is the interchange fee. Mastercard charges the vendor 1.35% of the sale, plus an additional percentage up to 3.25%, and a fixed $001 fee for each transaction.

Added to that is an assessment fee. This fee is 0.14% for Visa cards.

Calculating the Balance of a Credit Card

The **balance**, or sometimes balance due, on a credit card is the previous balance, plus all expenses, minus all payments and credits, plus the interest on the card. As stated before, if the card was paid off, there is no interest to be paid.

Example 5 **Calculating the Balance of a Credit Card**

Find the balance on the credit card with the given interest charge and balance before interest was charged. The cards were not paid off previously.

1. Balance before interest is $708.50, interest charge is $8.15
2. Balance before interest is $1,395.10, interest charge is $21.32

Solution

1. Adding the balance before interest to the interest charge, we find the balance to be $716.65.
2. Adding the balance before interest to the interest charge, we find the balance to be $1,416.42.

The next example puts all those steps together.

Example 6 **Find Balance Due from Transactions and Interest Rate**

Kaylen's credit card charges 16.9% annual interest. His current billing period is from November 1 to November 30. The balance on November 1 was $1,845.23. Use Kaylen's following transactions to determine his balance due at the end of the billing cycle.

Date	Activity	Amount
1-Nov	Billing Date Balance	$1,845.23
3-Nov	Groceries	$78.50
4-Nov	Tablet	$159.00
4-Nov	Online Game Purchase	$39.99
4-Nov	Restaurant	$47.10
10-Nov	Payment	$300.00
13-Nov	Gasoline	$58.75
13-Nov	Clothing	$135.00
18-Nov	Gift	$30.00
18-Nov	Restaurant	$21.75
28-Nov	Gasoline	$43.79

Solution

The first step is to find Kaylen's average daily balance. To find the average daily balance, we use the following steps.

1. Start with a list of transactions with their dates and amounts.

This list is provided.

1. For each day that had transactions, find the total of the transactions for the day. The days with more than one transaction were Nov. 4, Nov. 13, and Nov. 18. Treating the payment on November 10 as a negative value, the daily transaction amounts are

Date	Amount
1-Nov	$1,845.23
3-Nov	$78.50
4-Nov	$246.09
10-Nov	-$300.00
13-Nov	$193.75
18-Nov	$51.75
28-Nov	$43.79

2. Create a table containing each day with a different balance. The new table with dates that had different balances is below.

Date	Balance
1-Nov	\$1,845.23
3-Nov	\$1,923.73
4-Nov	\$2,169.82
10-Nov	\$1,869.82
13-Nov	\$2,063.57
18-Nov	\$2,115.32
28-Nov	\$2,159.11

3. Now, add a column for the number of days those balances until the balance changed. The days until the balance changes is found by finding the difference in the dates. For instance, from May 15 to May 26 was 11. Adding that column to the table we have The last entry was 3 since there are 30 days in November (the 28th, 29th, and 30th).

Date	Balance	Days Until Balance Changes
1-Nov	\$1,845.23	2
3-Nov	\$1,923.73	1
4-Nov	\$2,169.82	6
10-Nov	\$1,869.82	3
13-Nov	\$2,063.57	5
18-Nov	\$2,115.32	10
28-Nov	\$2,159.11	3

4. Add a column that contains the balances multiplied by the number of days until the balance changed. We create the column and multiply the values.

Date	Balance	Days Until Balance Changes	Balance Times Days
1-Nov	\$1,845.23	2	\$3,690.46
3-Nov	\$1,923.73	1	\$1,923.73
4-Nov	\$2,169.82	6	\$13,018.92
10-Nov	\$1,869.82	3	\$5,609.46
13-Nov	\$2,063.57	5	\$10,317.85
18-Nov	\$2,115.32	10	\$21,153.20
28-Nov	\$2,159.11	3	\$6,477.33

5. Find the sum of that last column. Adding that last column we have a sum of \$62,190.95.
6. There are 30 days in November, so divide the sum by 30, which gives an average of \$2,073.03, which is the average daily balance.

   With the average daily balance, we can determine the interest that is charged for November. Substituting $\text{ADB} = \$2{,}073.03$, $r = 0.169$, and $d = 30$ into the formula $I = \frac{\text{ADB} \times r \times d}{365}$ and calculating, we find the interest to be $I = \frac{\text{ADB} \times r \times d}{365} = \frac{\$\,2{,}073.03 \times 0.169 \times 30}{365} = \$\,28.80$.
   This interest is added to the final balance from the table in step 3, \$2,159.11, which yields a balance due of \$2,101.83.

Minimum Payment Due

The **minimum payment** due is the smallest required amount to be paid on a credit card to avoid late fees and penalties, such as an increased interest rate. The calculations for this may differ from card to card. They also depend in the balance of the credit card. General guidelines for minimum payment due are:

- For larger balances (usually over \$1,000), the minimum payment will be some percentage of the balance due.
- For moderate balances (between \$25 and \$1,000), the minimum would be a specified dollar amount. \$25 seems to be a common value.
- If the balance is small (under \$25 for instance), then the minimum payment is the balance.

  Those are just guidelines. Individual cards may vary in these values.

  Minimum payments should only be paid if money is short in a given month. The length of time to pay off a credit card using minimum payments is quite long, and results in paying a lot of interest. It is strongly discouraged.

Example 7 **Calculate the Minimum Payment Due**

The FYA credit card company has the following minimum payment policy. For balances over $1,000, the minimum payment is 2.5% of the balance due plus fees, but not interest. For balances between $500.00 and $999.99, the minimum payment is $50.00. For balances $499.99 and under, the minimum payment is $25.00 or the balance due, whichever is smaller.

In the following, calculate the minimum payment due given the credit card minimum payment policy, the balance due and fees charged.

1. Balance due is $1,309.00, no fees
2. Balance due is $265.50, $35 in fees
3. Balance due is $784.90, no fees

Solution

1. The balance is over $1,000, so the minimum payment is 2.5% of the balance due plus fees. 2.5% of the balance due is $0.025 \times \$\,1,309.00 = \$\,32.73$. Since there are no fees, the minimum payment due is $32.73.
2. The balance is under $499.99, so the minimum payment due is $25.00.
3. The balance is between $500.00 and $999.00, so the minimum payment due is $50.00.

Check out this nerdwallet article about minimum payments for more!

Key Terms

- Reward program
- Annual fee
- Credit limit
- Bank-issued credit card
- Store-issued credit card
- Travel and entertainment cards
- Charge cards
- Billing period
- Balance
- Minimum payment
- Average daily balance

Key Concepts

- Credit cards can be a flexible way to pay for almost anything, but can become a financial hazard if used unwisely.

- When deciding which credit card to apply for, evaluate the interest rate, fees (annual and late), reward programs and credit limit. Be sure they meet your criteria.
- Paying off the balance of your credit card every month will control your spending and will never result in paying interest.
- Credit card statements hold all important information about your credit card, including payment, balances, charges and billing cycle dates.
- Although the minimum payment is attractive precisely because it is so small, paying only the minimum results is a long payoff term and higher interest costs.

Video

- Choosing a Credit Card
- Reading Credit Card Statements

Formula

$$I = \frac{\text{ADB} \times r \times d}{365}$$

6.11 Buying or Leasing a Car

Figure 25: The choice to lease or buy is based on cost and other concerns.

Learning Objectives

After completing this section, you should be able to:

1. Evaluate basics of car purchasing.
2. Compute purchase payments and identify the related interest cost.
3. Evaluate the basics of leasing a car.
4. Identify and contrast the pros and cons of purchasing versus leasing a car.
5. Investigate the types of car insurance.

6. Solve application problems involving owning and maintaining a car.

   There are people who don't need a car and won't purchase one. But for many people, whether or not to have a car is not a question. Having a car is a basic necessity for these people.

   Obtaining a car can be daunting. The models, the features, the additional costs, and finding funding are all steps that need to be taken. One of the big decisions is whether to buy the car or to lease the car. This section will address some of the issues associated with each option.

The Basics of Car Purchasing

The biggest questions you will answer before purchasing a car are, what do you want and what do you need?

Does it have to be new? Does it have to be a make and model you are familiar with? Does it have to have assisted driving? What other details are important to you? For a new vehicle, every feature beyond standard features comes with additional cost, which leads to the question that constrains all of your decisions about a car. How much can you afford to spend on a car?

What you can afford must include insurance costs (discussed later in this section) and maintenance and upkeep. Once you have this in mind, you can search for a car that matches, as closely as possible, what you want and can afford. Most, if not all, dealers have websites that you can search through to identify the car you want. If new cars are not affordable, used cars cost less but come with the wear and tear of use.

The sticker price of the car, called the manufacturer's suggested retail price (MSRP), or the negotiated price you arrive at, isn't the end of the cost to buying a car. There are many fees that accompany the purchase of the car, and perhaps even sales tax. These include but aren't necessarily limited to the following:

- the **title and registration fee**, which includes registering your car with the state, getting the license plate, and assigning the title of the car to the lender. This cannot be avoided.
- a **destination fee**, which covers the cost of delivering the vehicle to the dealer
- a **documentation fee**, sometimes referred to a processing fee of handling fee, is the cost of all the paperwork the dealer did to get you the car
- a **dealer preparation fee**, which is for washing the car and other preparation of that sort. You should try to negotiate that out of the cost of the dealer tries to charge for that
- **extended warranties** and maintenance plans, which help cover some of the costs of caring for the car.
- Sales tax.

  You could pay for these immediately, but they are often added to the financing of the car, meaning they become part of the principal of your loan.

Example 1 **Total Cost to Purchase a Car**

Nichole negotiates with her car dealership so that the price is $21,800. She needs to pay the 6.75% sales tax on the car. Other fees are $31.00 for title and registration, $1,000 in destination fees, and a documentation fee of $175. What is the total cost of Nichole's car?

Solution

We add the car's sales cost, sales tax and all other fees to arrive at this value. The sales tax is 6.75% on the price she negotiated, so is $\$\,21,800 \times 0.0675 = \$\,1,471.50$. Adding these up, we have a total cost of $\$\,21,800 + \$\,1,471.50 + \$\,31.00 + \$\,1,000 + \$\,175 = \$\,24,477.50$.

One way to bring down payments on a car is to provide a **down payment** or a trade in. This is money applied to the purchase price before financing happens. Be warned, the sales tax applies to the full purchase price! If you reduce the amount financed, the payments respond by going down. This often becomes part of the negotiating process.

Example 2 **Total Cost to Purchase a Car with Down Payment**

Sophia negotiates a $19,800 price for her new car. The sales tax is 9.5% in her area, and the dealership charges her $300 in documentation fees. Her title, plates, and registration come to $321.50. The dealership adds to this a destination fee of $1,100. If she places a down payment of $5,000 on the car, what is the total she will finance for the car?

Solution

The price was $19,800. The sales tax of 9.5% is based on this number. The sales tax comes to $\$\,19,800 \times 0.095 = \$\,1,881$. Adding all the fees to the price and the sales tax brings the total cost of the car to $\$\,19,800 + \$\,1,881 + \$\,300 + \$\,321.50 + \$\,1,100 = \$\,23,402.50$. Her down payment is applied to this number, so the $5,000 is subtracted from $23,402.50. The subtraction yields the amount to be financed, which is $18,402.50.

When purchasing a car, the total cost to obtain the car is not the only factor in your monthly price. You will also pay an interest rate for the loan you obtain. The interest rate you will get is dependent on your credit score (see The Basics of Loans). But you can choose from different lenders. The dealership will likely offer to finance your car loan. Frequently, dealerships offer special financing with very low rates. This is to help move inventory, and may indicate their desire to make sales. This might make negotiating easier. Even if the dealership offers financing, check with your bank or credit union to determine the interest rates they are offering. To reduce your payments, choose the lowest rate you can find.

Purchase Payments and Interest

Whether or not you buy a new car or a used car, if you finance the purchase, you are taking out a loan. The interest rates available for used cards are frequently higher than those for new cars. These loan payments work exactly the same way as other loans do as far as payments are concerned. The payment function comes from The Basics of Loans. The difference between financing a new car or a used car is that financing a new car typically comes with a lower interest rate and a longer term that financing a used car.

FORMULA

The payment, pmt, per period to pay off a loan with beginning principal P is $pmt = \frac{P \times \left(\frac{r}{n}\right) \times \left(1+\frac{r}{n}\right)^{n \times t}}{\left(1+\frac{r}{n}\right)^{n \times t} - 1}$, where r is the annual interest rate in decimal form, t is the term in years, and n is the number of payments per year (typically, loans are paid monthly making $n = 12$). Note, payment to lenders is always rounded up to the next penny.

CHECKPOINT

Often, the formula takes the form $pmt = \frac{P \times (r) \times (1+r)^n}{(1+r)^n - 1}$, *where r is the interest rate per period (annual rate divided by the number of periods per year), and n is the total number of payments to be made.*

Example 3 **New Car Payments**

In the following, calculate the monthly payment using the given total to be financed, the interest rate, and the term of the car loan.

1. Total to be financed is \$31,885, interest rate is 2.9%, for 5 years.
2. Total to be financed is \$22,778, interest rate is 4.5%, for 6 years.

Solution

1. The amount to be financed is the principal, P, which is \$31,885. The rate r is 0.029, and the term is $t = 5$ years. These are monthly payments, so $n = 12$. Substituting and calculating, we find the monthly payment to be

$$\begin{aligned}
pmt &= \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n \times t}}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1} \\
&= \frac{\$\,31{,}885 \times \left(\frac{0.029}{12}\right) \times \left(1 + \frac{0.029}{12}\right)^{12 \times 5}}{\left(1 + \frac{0.029}{12}\right)^{12 \times 5} - 1} \\
&= \frac{\$\,31{,}885 \times \left(0.00241\overline{6}\right) \times \left(1.00241\overline{6}\right)^{60}}{\left(1.00241\overline{6}\right)^{60} - 1} \\
&= \frac{\$\,31{,}885 \times \left(0.00241\overline{6}\right) \times (1.15583736592)}{1.15583736592 - 1} \\
&= \frac{\$\,89.0635298302}{0.15583736592} \\
&= \$\,571.52
\end{aligned}$$

2. The amount to be financed is the principal, P, which is \$22,778. The rate r is 0.045, and the term is $t = 6$ years. These are monthly payments, so $n = 12$. Substituting and calculating, we find the monthly payment to be

$$\begin{aligned} pmt &= \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n \times t}}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1} \\ &= \frac{\$\,22{,}778 \times \left(\frac{0.045}{12}\right) \times \left(1 + \frac{0.045}{12}\right)^{12 \times 6}}{\left(1 + \frac{0.045}{12}\right)^{12 \times 6} - 1} \\ &= \frac{\$\,22{,}778 \times (0.00375) \times (1.00375)^{72}}{(1.00375)^{72} - 1} \\ &= \frac{\$\,22{,}778 \times (0.00375) \times (1.3093031051)}{1.3093031051 - 1} \\ &= \frac{\$\,111.837397673}{0.3093031051} \\ &= \$\,361.58 \end{aligned}$$

Example 4 **Used Car Payments**

Calculate the monthly payment for the used car if the total to be financed is \$16,990, the interest rate is 7.5%, and the loan term is 3 years.

Solution

The amount to be financed is the principal, P, which is \$16,990. The rate r is 0.075, and the term is $t = 3$ years. These are monthly payments, so $n = 12$. Substituting and calculating, we find the monthly payment to be

$$\begin{aligned} pmt &= \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n \times t}}{\left(1 + \frac{r}{n}\right)^{n \times t} - 1} \\ &= \frac{\$\,16{,}990 \times \left(\frac{0.075}{12}\right) \times \left(1 + \frac{0.075}{12}\right)^{12 \times 3}}{\left(1 + \frac{0.04775}{12}\right)^{12 \times 3} - 1} \\ &= \frac{\$\,16{,}990 \times (0.00625) \times (1.00625)^{36}}{(1.00625)^{36} - 1} \\ &= \frac{\$\,16990 \times (0.00375) \times (1.25144613551)}{1.25144613551 - 1} \\ &= \frac{\$\,132.887936515}{0.25144613551} \\ &= \$\,528.50 \end{aligned}$$

The Basics of Leasing a Car

Leasing a car is an alternative to purchasing a car. It is still a loan, and acts like one in many respects. They typically last either 24 months or 36 months, though other terms are available. Leases also come with mileage limits, frequently 10,000, 12,000, or 15,000 miles per year. When the lease is over, the car is returned to the dealer. At that time there may be fees that have to be paid, such as for damage to the car or for extra miles driven over the limit.

There are two components to lease costs. One is the monthly payment for the lease. The other is the fees for leasing, These often are paid before the lease is complete. These include:

- a down payment, which is your initial payment that is applied to the price of the car. It reduces the amount you finance, much the same as when you purchase a car. It is recommended that this be negotiated away.
- the **acquisition fee**, sometimes called the bank fee. This is the money charge for the company to set up the lease. It is essentially a paperwork fee. It is not likely that this can be negotiated.
- a **security deposit**, which might be required. It is about the same as 1 month's payment for the lease. The deposit is returned to you if the car is in good shape at the end. This can be negotiated away.
- **disposition fees**, which cover the cost the company will incur when they take your car back and are typically between $200 and $450.
- the title, registration, and license fees, just as with the purchase of a car.
- sales tax, which will likely be applied. The sales tax only covers the depreciated portion of the car (more on depreciation later) in many states. Since this depends on the state in which the car is leased, you should determine the sales tax rules for where you lease the car.

  As you can imagine, this can come to a fairly high dollar amount.

Example 5 **Cost to Obtain a Lease**

Donna wants to lease a Subaru Outback in Eden, New York. Find the total cost of obtaining her lease if there is no down payment, $175.00 in acquisition fees, a security deposit of $300.00, $350.00 in disposition fees, $102.50 in title and registration fees, and sales tax of $3,536.05.

Solution

Adding these values together we find the total cost is $4,463.55.

PEOPLE IN MATHEMATICS

Zollie Frank

Zollie Frank and Armund Shoen founded one of the original leasing companies, Four Wheels, in 1939. Their company leased automobiles to corporations. They began by leasing 5 cars to the Petrolager pharmaceutical company in year one. This saved Petrolager money and provided a steady cash flow to the Four Wheels business. In year two, the number of cars leased to Petrolager was 75. Their new idea was to lease cars directly to companies for one year. Previously, such companies might pay for mileage, gas, and a partial down payment. Sadly, the salesmen who were being so helped often left the company before the car was paid for, and so the company lost the down payment money.

The lease was for $45 per month per car for one year.

You have some obligations when you lease a car. You must keep the car in good condition, cleaned, maintained, and free of anything more than minor damage. If the car is in poor condition when the car is returned, you will be responsible for the cost to bring the car to an acceptable condition. You are also expected to keep the mileage under its limit. If you go over, you will pay 10 to 25 cents per mile over.

Lease Payments

Lease payments are similar to regular loan payments, but have some other details. Calculating a lease payment involves knowing the following values:

- **The price of the car.** This is the cost you would pay for the car after applying all discounts, incentives, and negotiations.
- **Residual Value.** This is the manufacturer's estimate of the car's value after a set period of time. The residual value is expressed as a percentage of the manufacturer's suggested retail price (MSRP).
- **Months.** This is the length of the lease. Most leases are either 24- or 36-month leases, but other terms are available.
- **Monthly Depreciation**. The monthly depreciation is the difference between the price of the car and the residual value, divided by the number of months of the lease, and represents the monthly loss of value of the car while it's being used.

- **Money Factor (MF).** This is the interest rate, but expressed in a different way for a lease. Converting from the money factor to the annual percentage rate (APR) is done by multiplying the MF by 2400. Naturally, converting an APR to a MF is done by dividing the APR by 2400.

FORMULA

The monthly depreciation for a car, MD, is $\text{MD} = \frac{P-R}{n}$, P is the price paid for the car, R is the residual value of the car, and n is the number of months of the lease.

The annual percentage rate for a lease is $\text{APR} = 2400 \times \text{MF}$, where MF is the money factor of the lease. The MF for a lease is $\text{MF} = \frac{\text{APR}}{2}, 400$.

Example 6 **Monthly Depreciation of a Car**

1. The purchase price of a car is $25,000. Its residual price is $14,500. What is its monthly depreciation for a 36-month lease?
2. The purchase price of a car is $30,000. Its residual price is $18,600. What is its monthly depreciation for a 24-month lease?

Solution

1. The monthly depreciation formula is $\text{MD} = \frac{P-R}{n}$, Substituting $25,000 for P, $14,500 for R, and 36 for n, we find MD to be $\text{MD} = \frac{P-R}{n} = \frac{\$\,25{,}000 - \$\,14{,}500}{36} = \frac{\$\,10{,}500}{36} = \$\,291.67$.
2. The monthly depreciation formula is $\text{MD} = \frac{P-R}{n}$, Substituting $30,000 for P, $18,600 for R, and 24 for n, we find MD to be $\text{MD} = \frac{P-R}{n} = \frac{\$\,30{,}000 - \$\,18{,}600}{24} = \frac{\$\,11{,}400}{24} = \$\,475.00$.

Example 7 **Converting Between APR and MF**

1. Find the annual percentage rate if the money factor is 0.00001875.
2. Find the money factor if the APR is 6.25%.

Solution

1. The APR is the money factor times 2400, so $\text{APR} = 2400 \times \text{MF} = 2400 \times 0.00001875 = 0.045$. Expressed as a percentage, the APR is 4.5%.
2. The MF is the APR divided by 2400, so $\text{MF} = \frac{\text{APR}}{2400} = \frac{0.0625}{2400} = 0.000026041\overline{6}$.

Once the values above are found, the payment for the lease can be calculated.

FORMULA

The payment, PMT, for a lease is $PMT = \frac{(P-R)}{n} + (P+R) \times \text{MF}$, where P is the price paid for the car, R is the residual value of the car, n is the number of months of the lease, and MF is the money factor for the lease.

Example 8 **Calculating Car Lease Payments**

Calculate the lease payments for car with the following price, residual price, length of lease, and money factor or APR.

1. Price is $28,344, residual price is $18,140.16, 24-month lease, money factor is 0.000025.
2. Price is $22,500, residual price is $13,050, 36-month lease, APR is 7.5%.

Solution

1. Substituting the values $P = \$28{,}344$, $R = \$18{,}140.16$, $n = 24$ and $\mathrm{MF} = 0.000025$ into the formula and calculating, the monthly lease payment is

$$\begin{aligned} PMT &= \frac{(P-R)}{n} + (P+R) \times \mathrm{MF} \\ &= \frac{(\$\,28,344 - \$\,18,140.16)}{24} + (\$\,28,344 + \$\,18,140.16) \times 0.000025 \\ &= \frac{\$\,10{,}203.84}{24} + (\$\,46,484.16) \times 0.000025 \\ &= \$\,426.33 \end{aligned}$$

2. Given the APR, we find the MF which is $\mathrm{MF} = \frac{\mathrm{APR}}{2400} = \frac{0.075}{2400} = 0.00003125$. Substituting the values $P = \$28{,}344$, $R = \$18{,}140.16$, $n = 24$ and the MF into the formula and calculating, the monthly lease payment is

$$\begin{aligned} PMT &= \frac{(P-R)}{n} + (P+R) \times \mathrm{MF} \\ &= \frac{(\$\,22,500 - \$\,13,050)}{36} + (\$\,22,500 + \$\,13,050) \times 0.00003125 \\ &= \frac{\$\,9{,}450}{24} + (\$\,35,550) \times 0.000025 \\ &= \$\,263.62 \end{aligned}$$

Comparing Purchasing and Leasing

When deciding to buy or lease a car, the differences between the two options should be carefully evaluated. The following is a list of points of comparison between the two.

- The payments for a lease are likely less than the payment for purchasing.
- When leasing, you get a new car after the lease term is over, typically 24 or 36 months. Buying the car means the same car is driven until it is re-sold and a new one bought. Essentially, leasing a car is equivalent to renting a car.
- The leased car is new, so all warrantees are in force and you drive the car during its best years. When the car is purchased, it may be kept past its warrantees and may be driven until it is quite old.
- Each time you lease a new car, all the fees and taxes must be paid again. When buying a car, these fees are only paid once.

- Leasing contracts carry restrictions on the mileage you can drive per year, and going over incurs more cost at the end of the lease. Buying the car means no such mileage limits.
- When leasing, you are obligated to keep the vehicle in good condition and maintained according to the dealer's schedule. Some dealerships will even pay for oil changes over the life of the lease. When the car is purchased, the upkeep schedule is the choice of the owner.
- When a car is purchased and kept for long enough, the warranty expires and the owner is responsible for all maintenance items and repairs. The warrantee for a car won't expire during the lease term.
- When a new car is purchased and the loan is paid off the car is still owned by the buyer and may be traded in when a new car is to be purchased. When leasing, the car is returned to the dealer when the lease term is over.

  When deciding between the two, you are choosing between these features. If you aren't willing to drive an older car or deal with the upkeep that accompanies an older car, you may want to lease. This means you will need to pay those beginning costs each time the lease is up. If you want to own the car after the payments are over, then you may want to buy a car. This means you are paying for all the upkeep after the warrantees expire, but you have no limits on mileage and own the car at the end. It really depends on your preferences.

Example 9 **Lease or Buy**

In the following, determine if a lease or purchase of a car is better.

1. Joyce is concerned with large repairs and does not want to deal with them.
2. Maurice prefers to drive newer cars.

Solution

1. Since Joyce does not want to deal with repairs, so leasing would be a better choice. This way, the warranty covers most of the big repairs that could need to be done.
2. Since Maurice likes to drive new cars, leasing is a better option, since he will lease a new car every 2 to 3 years.

Car Insurance

Car insurance is meant to cover costs associated with accidents involving cars. Most states (all except New Hampshire and Virginia) require some insurance. Without insurance, the state may not let you get a license for your car or register your car. Your state's requirements can be hard to follow. Fortunately, insurance companies and brokers will make sure your insurance is sufficient for your state and will warn you if you try to not meet the requirements. Of course, they may offer more than what is sufficient, so it is your responsibility to determine how much coverage you want, as long as the minimum insurance requirements are met. The cost of insurance should be accounted for when evaluating the affordability of buying or leasing a car.

Whether your car is leased or owned, you do need insurance. This contributes to the cost of having the vehicle. Leasing or owning makes no difference to the insurance company you choose, because they are insuring you based on what you are driving, your driving record, and other information about you including where you live and your age. These insurance policies have many components that address different costs that can come from auto accidents. This may make details confusing, and you may not realize what you are paying for until you must use it. Here is a brief outline of the different components of auto insurance, many of which are required by the state that issues your driver's license.

- **Liability insurance** is mandatory coverage in most states. Liability insurance covers property damage and injuries to others should you be found legally responsible for an accident. You are required to have the minimum amount of coverage, as determined by your state, in both areas.
- **Collision insurance** is insurance covering the damage caused to your car if involved in an accident with another vehicle.
- **Comprehensive insurance** is an extra level of coverage if involved in an accident with another vehicle and covers other things like theft, vandalism, fire, or weather events as outlined in your policy. There is a deductible assigned to each type of insurance, an amount that you pay out of pocket before your comprehensive coverage takes effect. Comprehensive insurance is often required if you lease or finance the purchase of a vehicle.
- **Uninsured or underinsured motorist insurance:** If you are hit by an uninsured or underinsured motorist, this insurance will help pay medical bills and damage to your car.
- **Medical payments insurance** is mandatory in some states and helps pay for medical costs associated with an accident, regardless of who is at fault.
- **Personal injury protection insurance** is coverage for certain medical bills and other expenses due to a car accident. Other covered expenses may include loss of income or childcare, depending on your policy.
- **Gap insurance** is designed to cover the gap between what is owed on the car and what the car is worth in the event your car is a total loss.
- **Rental reimbursement insurance** is coverage for a rental car while your car is under repair resulting from an accident.

  You can also purchase other special insurance policies, such as classic car insurance, new car replacement insurance, and sound system replacement insurance, to name a few. It is important that you determine exactly what you need, as insurance policies can be expensive and vary according to your age, driving history, and where you live.

Example 10 **Types of Insurance**

1. Which component of insurance pays if you are in an accident with a motorist without insurance?
2. Which component of insurance pays for the remaining principal owed on your car in the case of a total loss?

Solution

1. Uninsured motorist insurance covers accidents with those who have no insurance.
2. Gap insurance will cover the gap between what is owed on the car and what it is worth if an accident results in a total loss.

Example 11 **Monthly Cost of Owning a Car**

If your car payment is \$287.50 per month and your car insurance is \$930 every 6 months, what is the cost of the car per month when accounting for the insurance?

Solution

The cost of the car including insurance is the monthly payment, \$287.50, plus the monthly cost of the insurance. The insurance cost per month is $\$\,\frac{930}{6} = \$\,155$ since the insurance cost is for every 6 months. Adding those the cost with insurance is \$442.50.

Maintaining a Car

Cars are not a buy it and forget it item. They require upkeep, which adds to the cost of owning the car. Tires, brakes, and wipers need replacing. Oli changes, inspections, so many things other than gasoline. Below is a list of some maintenance requirements for cars, along with cost and roughly how often they should happen.

Maintenance	Frequency	Cost Range
New Tires	Every 5 years	\$25–\$300 per tire
Oil Change	Every 3,000–6,000 miles	\$35–\$75
Wipers	Every 6–12 months	\$20–\$40
Inspection	Annual	\$10–\$50
Brake pads	10,000–20,000 miles	\$200–\$300
Air Filter	15,000–30,000 miles	\$35–\$80

When designing a budget, these expected costs should be accounted for. Extra money per month should be saved in addition to this budget category, to handle unanticipated, and perhaps very costly, repairs.

Example 12

Estella needs to budget for her car maintenance. She expects to buy new tires each 4 years, which will cost her \$480 to replace them all. Oil changes near her cost \$49.99, and she believes she will get one every 4 months. Her inspection costs \$15 per year. Wipers for her car cost \$95 for all three and she anticipates changing them every year. She drives less than 30,000 miles per year, so she plans to replace the air filter once per year. The air filter for her car costs \$57.50. How much should she budget per month to cover these costs?

Solution

Her yearly costs are the wipers, inspection, and tires, which total \$167.50. Tires will be bought every 5 years, so per year she should budget \$96. Her oil changes, which will happen three times per year, cost \$49.99 each, so she'll spend \$149.97 for the year on oil changes. Adding these up, her yearly budget should include \$413.47 for maintenance. Dividing by 12 gives the monthly budget for maintenance, which is \$34.46 (rounded up to the next penny).

Key Terms

- Title and registration fees
- Destination fee
- Documentation fee
- Dealer preparation fee
- Extended warranty
- Down payment
- Acquisition fee
- Security deposit
- Disposition fees
- Liability insurance
- Collision insurance
- Comprehensive insurance
- Uninsured or underinsured motorist insurance
- Medical payment insurance
- Personal injury insurance
- Gap insurance
- Rental reimbursement insurance

Key Concepts

- There are many factors to consider when choosing to buy or lease a car.
- The cost of the car is increased by a number of fees and sales tax.
- There are advantages to buying a car and advantages to leasing a car. The decision between the two depends on the preference of the buyer.
- Insurance covers costs associate with accidents. It is made up of various components.
- The costs of owning a car, including insurance and maintenance, should be a part of the budgeting process.

- Budgeting for unexpected repairs can ease the stress of encountering large repair bill.

Formula

$$pmt = \frac{P \times \left(\frac{r}{n}\right) \times \left(1 + \frac{r}{n}\right)^{n\times t}}{\left(1 + \frac{r}{n}\right)^{n\times t} - 1}$$

$$\text{MD} = \frac{P - R}{n}$$

$$\text{APR} = 2400 \times \text{MF}$$

$$\text{MF} = \frac{\text{APR}}{2}, 400$$

$$PMT = \frac{(P - R)}{n} + (P + R) \times \text{MF}$$

6.12 Renting and Homeownership

Figure 26: Purchasing a home is a big investment, while renting is a lower cost alternative.

Learning Objectives

After completing this section, you should be able to:

1. Evaluate advantages and disadvantages of renting.
2. Evaluate advantages and disadvantages of homeownership.
3. Calculate the monthly payment for a mortgage and related interest cost.
4. Read and interpret an amortization schedule.
5. Solve application problems involving affordability of a mortgage.

After renting an apartment for 10 years, you realize that it may be time to purchase a home. Your job is stable, and you could use more space. It is time to investigate becoming a homeowner. What are the things that you must consider, and what is the financial benefit of owning as opposed to renting? This section is about the advantages, disadvantages, and costs of homeownership as opposed to renting.

Advantages and Disadvantages of Renting

When renting, you will likely sign a **lease**, which is a contract between a renter and a landlord. A **landlord** is the person or company that owns property that is rented. The lease will detail your responsibilities, restrictions on activities, deposits, fees, maintenance, repairs, and rent during the term of the lease. It also defines what your landlord can, and cannot, do with the property while you occupy the property.

Like leasing a car, there are advantages to renting but also some disadvantages. Some advantages are:

- Lower cost.
- Short-term commitment.
- Little to no maintenance cost. The landlord pays for or performs most maintenance.
- You need not stay at end of lease. Once the lease term is over (the lease is up), you are not obligated to stay.
- If renting in an apartment complex, there may be a pool, gym, or community room for renters to use.

  Of course, there are disadvantage too:

  - No tax incentives.
- Housing cost is not fixed. When the lease is up, the rent can change.
- No equity. When you are done living in a rental, you have built no value.
- Restrictions on occupants. There may be a limit on how many can live in the apartment.
- Restrictions on decorating. The property is not yours, so any decorating or improvements need landlord permission.
- Limits on pets. Permission for pets, and their number and type, will be set forth in the lease.
- May not be able to remain when lease term is over. The landlord can, at the end of your lease, invite you to leave.
- The building may be sold, and the new landlord may institute changes to the lease when the previous lease expires.

  Renting has fees to be paid at the start of the lease. Typically, when you rent, you will pay first and last months' rent and a security deposit. A security deposit is a sum of money that the landlord holds until the renter leaves the rental property. The deposit will cover repairs for damage to the apartment during the renter's stay but may be returned if the apartment is in good condition. If your landlord runs a credit check on you, the landlord may charge you for that.

Advantages of Buying a Home

The advantages to buying a home mirror the disadvantages of renting, and the disadvantages of home ownership mirror the advantages of renting.

Some advantages to buying a home are:

- There are tax incentives. The interest you pay for your mortgage (more on that later) is deductible on your federal income tax.
- There are no restrictions on pets or occupants, unless laws in your area specify limits for homes.
- You can redecorate any way you wish, limited only by the laws in your area.
- Once your mortgage is set with a fixed-interest rate, your housing cost is fixed.
- Your home grows equity, that is, the difference between what you owe and what the house is worth grows. You can use the equity to secure loans, and you recover the equity (and more if you're fortunate) when you sell the house.
- As long as you pay your mortgage and maintain the home to the standards of your community, you can stay as long as you wish.

  Some disadvantages to home ownership are:

  - The cost is higher than renting. Mortgages and associated costs are typically higher than rent for a similar living space.
- The owner is responsible for upkeep, maintenance, and repairs. These can be extremely costly.
- The owner cannot walk away from the property. It can be sold, but simply leaving the property, especially if not paid off yet, has serious consequences.

  The big question of affordability looms large over the decision to rent or buy. Renting, strictly from an affordability viewpoint, comes with much less initial outlay and smaller commitment. If you do not have sufficient income to regularly save for possibly expensive repairs, or your credit isn't quite as good as it needs to be, then renting may be the best choice. Of course, even if you can afford to buy a home, you may choose to rent based on the comparative advantages.

> **VIDEO**
> Rent or Buy

Buying a home really involves two buyers. You and the mortgage company. The mortgage company has interest in the home, as they are providing the funds for the home. They want to protect their investment, and many fees are about the bank as much as the buyer. They fund a mortgage based on the value they assign the property. Not you. This means they will want some certainty that the home is sound, and you are a good investment.

WHO KNEW?

Closing Costs

When a home is bought, there are many costs that need to be paid at the time of purchase, which are lumped under the term closing costs. At the start of 2022, the average closing costs for a single-family home exceeded $6,800. These costs include:

- The appraisal fee, which is what is paid to someone to establish the home's worth. The value of the home to the bank may differ from what the home is listed for, or what an app tells you the home is worth. It may run approximately $350.
- The home inspection fee. The inspection should reveal any problems with the house that will need to be fixed either before or after you obtain the home.
- The title search. The is a records search to insure there are no issues with who actually owns the property. It can cost about 0.5% to 1% of the amount you are financing.
- Prepaid taxes. You will need to pay about 6 months of taxes at the time of purchase.
- The credit report fee. This is a fee for checking your credit. You might pay $25 or more for this.
- The origination fee. This is the price the mortgage company charges you to cover the costs of creating the mortgage. This could be 0.5% to 1% (or more) of the amount you are borrowing.
- The application fee. This is just a processing fee and could come to several hundred dollars.
- The underwriting fee. This covers the cost of verifying your financial qualifications. It could be a flat fee, or some small percentage of the amount financed. Such as 0.5% or 1%.
- Attorney fees. If you use an attorney, you will have to pay the attorney.
- State of local fees. This may include a filing fee charged by the county or municipality in which you reside.

  That's a long list, and it is not even complete. When buying, be prepared to see these costs. It can be surprising. But in the end, you will have equity in the home, which means when you sell your home, you will get some of your money back.

VIDEO

Closing Costs

In the end, you must weigh your options and carefully consider your priorities in choosing to rent or buy a home.

Mortgages

Some people will purchase a home or condo with cash, but the majority of people will apply for a mortgage. A **mortgage** is a long-term loan and the property itself is the security. The bank decides the minimum down payment (with your input), the payment schedule, the duration of the loan, whether the loan can be assumed by another party, and the penalty for late payments. The title of the home belongs to the bank.

Since a mortgage is a loan, everything about loans from The Basics of Loans holds true, including the formula for the payments.

Monthly Mortgage Payments

The formula to calculate your monthly payments of principal and interest uses APR as the annual interest rate.

FORMULA

The payment, pmt, per month to pay down a mortgage with beginning principal P is $pmt = \frac{P\times\left(\frac{r}{12}\right)\times\left(1+\frac{r}{12}\right)^{12\times t}}{\left(1+\frac{r}{12}\right)^{12\times t}-1}$, where r is the annual interest rate in decimal form and t is the number of years of the payment.

Note, payment to lenders is always rounded up to the next penny.

To find the total amount of your payments over the life of the loan, multiply your monthly payments by the number of payments.

Example 1 **30-Year Mortgage at 4.8% Interest**

Evan buys a house. His 30-year mortgage comes to \$132,650 with 4.8% interest. Find Evan's monthly payments.

Solution

Using the information above, $P = \$132{,}650$, $r = 0.048$ and $t = 30$. Substituting those values into the formula $pmt = \frac{P\times\left(\frac{r}{12}\right)\times\left(1+\frac{r}{12}\right)^{12\times t}}{\left(1+\frac{r}{12}\right)^{12\times t}-1}$ and calculating, we find the payment is

$$\begin{aligned}
pmt &= \frac{P\times\left(\frac{r}{12}\right)\times\left(1+\frac{r}{12}\right)^{12\times t}}{\left(1+\frac{r}{12}\right)^{12\times t}-1}\\
&= \frac{\$\,132{,}650\times\left(\frac{0.048}{12}\right)\times\left(1+\frac{0.048}{12}\right)^{12\times 30}}{\left(1+\frac{0.048}{12}\right)^{12\times 30}-1}\\
&= \frac{\$\,132{,}650\times(0.004)\times(1.004)^{360}}{(1.004)^{360}-1}\\
&= \frac{\$\,2{,}233.07781448}{3.20858992551}\\
&= \$\,695.97
\end{aligned}$$

His mortgage payment is \$695.97.

To find the total amount of your payments over the life of the loan, multiply your monthly payments by the number of payments. This can be useful information, but not too many people reach the end of their mortgage. They tend to move before the mortgage is paid off.

FORMULA

The total paid, T, on an t year mortgage with monthly payments pmt is $T = pmt \times 12 \times t$.

Example 2 **30-Year Mortgage at 5.35% Interest**

Cassandra buys a house. Her 30-year mortgage comes to $99,596 with 5.35% interest. If Cassandra pays off the mortgage over those 30 years, how much will she have paid in total?

Solution

To find the total paid over the life of the mortgage, use the formula $T = pmt \times 12 \times t$. To calculate this, the payment must be found. Using the information above, $P = \$99{,}596$, $r = 0.0535$ and $t = 30$. Substituting those values into the formula $pmt = \frac{P \times \left(\frac{r}{12}\right) \times \left(1 + \frac{r}{12}\right)^{12 \times t}}{\left(1 + \frac{r}{12}\right)^{12 \times t} - 1}$ and calculating, we find the payment is

$$\begin{aligned} pmt &= \frac{P \times \left(\frac{r}{12}\right) \times \left(1 + \frac{r}{12}\right)^{12 \times t}}{\left(1 + \frac{r}{12}\right)^{12 \times t} - 1} \\ &= \frac{\$\,99{,}596 \times \left(\frac{0.0535}{12}\right) \times \left(1 + \frac{0.0535}{12}\right)^{12 \times 30}}{\left(1 + \frac{0.048}{12}\right)^{12 \times 30} - 1} \\ &= \frac{\$\,99.596 \times \left(0.004458\overline{3}\right) \times \left(1.004458\overline{3}\right)^{360}}{\left(1.004458\overline{3}\right)^{360} - 1} \\ &= \frac{\$\,2{,}202.45911222}{3.96013414693} \\ &= \$\,556.16 \end{aligned}$$

Using the mortgage payment of $556.16 and $t = 30$ years in the formula $T = pmt \times 12 \times t$, the total that Cassandra will pay for the mortgage is $200,217.60.

With the principal of the mortgage and how much total is paid over the life of the mortgage, the cost of financing can be found by subtracting the principal of the mortgage from the total paid over the life of the mortgage.

FORMULA

The cost of financing a mortgage, CoF, is $\text{CoF} = T - P$ where P is the mortgage's starting principal and T is the total paid over the life of the mortgage.

Example 3 **30-Year Mortgage at 5.35% Interest**

Cassandra buys a house. Her 30-year mortgage comes to $99,596 with 5.35% interest. What was Cassandra's cost of financing?

Solution

In Example 2, we found that the total Cassandra will pay for the $99,569 mortgage is $200,217.60. Subtracting those we find the cost of financing $\text{CoF} = T - P = \$\,200,217.60 - \$\,99,596 = \$\,100,621.60$.

WHO KNEW?

Private Mortgage Insurance (PMI)

When you purchase a home, you will have to pay a down payment. This means you have money tied to the property, which lenders believe makes you less likely to walk away from a property. The amount of the down payment will be decided between you and the mortgage company. However, if your down payment is less than 20% of the property value, you will be required to pay private mortgage insurance (PMI). This is insurance you pay for so that the mortgage company is protected if *you* default on the loan. It often comes to between 0.5% and 2.25% of the original loan amount. It increases your monthly payment. Once you reach 20% of the loan value, you can request that the PMI be dropped. Even if you do not request cancelling the PMI, it will eventually and automatically be dropped.

For more, see this article on ways to get rid of PMI.

Reading and Interpreting Amortization Tables

Amortization tables were addressed in The Basics of Loans. They are most frequently encountered when analyzing mortgages.

The amortization table for a 30-year mortgage is quite long, containing 360 rows. A full table will not be reproduced here. We can, though, read information from a portion of an amortization table.

Example 4 **Amortization Table for a 30-Year, $165,900 Mortgage**

shows a portion of an amortization table for a 30-year, $165,900 mortgage. Use that table to answer the following questions.

1. What is the interest rate?
2. How much are the payments?
3. How much of payment 175 goes to principal?
4. How much of payment 180 goes to interest?
5. What's the remaining balance on the mortgage after payment 170?

Amortization Schedule Calculator					
Loan Amount		$165,900.00			
Interest Rate (annual)		5.61%			
Term (Years)		30			
Monthly Payment		$953.44			
Month	Payment	Principal	Interest	Total Interest	Balance
167	$953.44	$385.77	$567.67	$114,366.57	$121,041.52
168	$953.44	$387.57	$565.87	$114,932.44	$120,653.95
169	$953.44	$389.39	$564.06	$115,496.50	$120,264.56
170	$953.44	$391.21	$562.24	$116,058.73	$119,873.35
171	$953.44	$393.04	$560.41	$116,619.14	$119,480.32
172	$953.44	$394.87	$558.57	$117,177.71	$119,085.44
173	$953.44	$396.72	$556.72	$117,734.44	$118,688.73
174	$953.44	$398.57	$554.87	$118,289.31	$118,290.15
175	$953.44	$400.44	$553.01	$118,842.31	$117,889.71
176	$953.44	$402.31	$551.13	$119,393.45	$117,487.41
177	$953.44	$404.19	$549.25	$119,942.70	$117,083.22
178	$953.44	$406.08	$547.36	$120,490.06	$116,677.14
179	$953.44	$407.98	$545.47	$121,035.53	$116,269.16
180	$953.44	$409.89	$543.56	$121,579.09	$115,859.27
181	$953.44	$411.80	$541.64	$122,120.73	$115,447.47
182	$953.44	$413.73	$539.72	$122,660.45	$115,033.75
183	$953.44	$415.66	$537.78	$123,198.23	$114,618.09
184	$953.44	$417.60	$535.84	$123,734.07	$114,200.48
185	$953.44	$419.56	$533.89	$124,267.96	$113,780.93
186	$953.44	$421.52	$531.93	$124,799.88	$113,359.41
187	$953.44	$423.49	$529.96	$125,329.84	$112,935.92
188	$953.44	$425.47	$527.98	$125,857.81	$112,510.45
189	$953.44	$427.46	$525.99	$126,383.80	$112,082.99

Figure 27: Amortization table

Solution

1. Reading at the top of the table, we see the interest rate is 5.61%.
2. Reading from the top of the table or from the column labeled Payment, we see the payments are $953.44 per month.
3. In the row for payment 175, we see that the amount that goes to principal is $400.44.
4. In the row for payment 180, we see that the amount that goes to interest is $543.56.
5. In the row for payment 170, we see the remaining balance is $119,873.35.

Escrow Payments

The last few examples have looked at mortgage payments, which cover the principal and interest. However, when you take out a mortgage, the payment is sometimes much higher than that. This is because your mortgage company also has you pay into an **escrow account**, which is a savings account maintained by the mortgage company.

Your insurance payments will be set by your insurer and the mortgage company will pay them on time for you from your escrow account. Your property taxes are set by where you live and are typically a percentage of your property's **assessed value**. The assessed value is the estimation of the value of your home and does not necessary reflect the purchase or resale value of the home. Your property taxes will also be paid on time by the mortgage company from your escrow account.

For example, in Kalamazoo, Michigan, the effective tax rate for property is 1.69% of the assessed value of the home. These escrow payments, which cover bills for the home, can increase the monthly payments for your home well beyond the basic principal and interest payment.

Example 5 **Adding Escrow Payments to Mortgage Payments**

Jenna decides to purchase a home, with mortgage of \$108,450 at 6% interest for 30 years. The assessed value of her home is \$75,600. Her property taxes come to 5.7% of her assessed value. Jenna also has to pay her home insurance every 6 months, which is \$744 per six months. How much, including escrow, will Jenna pay per month?

Solution

Using the payment function to find her mortgage payments, $pmt = \frac{P\times\left(\frac{r}{12}\right)\times\left(1+\frac{r}{12}\right)^{12\times t}}{\left(1+\frac{r}{12}\right)^{12\times t}-1}$, with $P = \$108{,}405$, $r = 0.06$, and $t = 30$, her payments are

$$\begin{aligned} pmt &= \frac{P\times\left(\frac{r}{12}\right)\times\left(1+\frac{r}{12}\right)^{12\times t}}{\left(1+\frac{r}{12}\right)^{12\times t}-1} \\ &= \frac{\$\ 108{,}405\times\left(\frac{0.06}{12}\right)\times\left(1+\frac{0.06}{12}\right)^{12\times 30}}{\left(1+\frac{0.06}{12}\right)^{12\times 30}-1} \\ &= \$\ 649.95 \end{aligned}$$

Jenna also pays into escrow 1/12 of her property taxes per month. Her property taxes are 5.7% of the assessed value of \$75,600, which comes to $0.057 \times \$\ 75,600 = \$\ 4309.20$. This is an annual tax, so she pays 1/12 of that each month, or \$359.10. Jenna's home insurance is \$744 per 6 months, so each month she pays \$124.00 for insurance. Adding these together, her monthly payment is $\$\ 649.95 + \$\ 359.10 + \$\ 124.00 = \$\ 1,133.05$. This is quite a bit more than the \$649.95 for the principal and interest.

Key Terms

- Lease
- Landlord
- Mortgage
- Escrow account
- Assessed value

Key Concepts

- There are many points of comparison between renting and buying a house.

- Before deciding to buy a house, you should carefully consider all the responsibilities that come with home ownership.
- Renting comes with more restrictions on the renter, but with fewer costs and is easier to move from.
- Owning a house has more costs but has more freedom, plus the owner creates equity.
- Mortgages are loans, and payments are calculated in the same way as any other loan.
- Amortization tables help a homeowner understand the mortgage and how the payments are applied to the principal and interest.
- In addition to paying the amount financed for a mortgage, the monthly payment will include an escrow payment, which covers insurance and taxes.

Video

- Rent or Buy
- Closing Costs

Formula

$$pmt = \frac{P \times \left(\frac{r}{12}\right) \times \left(1 + \frac{r}{12}\right)^{12 \times t}}{\left(1 + \frac{r}{12}\right)^{12 \times t} - 1}$$

$$T = pmt \times 12 \times t$$

$$\mathrm{CoF} = T - P$$